PART ONE

A Library Of Elementary Functions
INTRODUCTION

We begin by discussing some algebraic methods for solving equations and inequalities. Next, we introduce coordinate systems that allow us to explore the relationship between algebra and geometry. Finally, we use this algebraic–geometric relationship to find equations that can be used to describe real-world data sets. We also consider many applied problems that can be solved using the concepts discussed in this chapter.
Section 1-1 LINEAR EQUATIONS AND INEQUALITIES

- Linear Equations
- Linear Inequalities
- Applications

The equation

$$3 - 2(x + 3) = \frac{x}{3} - 5$$

and the inequality

$$\frac{x}{2} + 2(3x - 1) \geq 5$$

are both first degree in one variable. In general, a first-degree, or linear, equation in one variable is any equation that can be written in the form

$$ax + b = 0 \quad a \neq 0$$

If the equality symbol, $=$, in (1) is replaced by $<$, $>$, $\leq$, or $\geq$, the resulting expression is called a first-degree, or linear, inequality.

A solution of an equation (or inequality) involving a single variable is a number that when substituted for the variable makes the equation (or inequality) true. The set of all solutions is called the solution set. When we say that we solve an equation (or inequality), we mean that we find its solution set.

Knowing what is meant by the solution set is one thing; finding it is another. We start by recalling the idea of equivalent equations and equivalent inequalities. If we perform an operation on an equation (or inequality) that produces another equation (or inequality) with the same solution set, then the two equations (or inequalities) are said to be equivalent. The basic idea in solving equations and inequalities is to perform operations on these forms that produce simpler equivalent forms, and to continue the process until we obtain an equation or inequality with an obvious solution.

**Linear Equations**

Linear equations are generally solved using the following equality properties:

**THEOREM 1 EQUALITY PROPERTIES** An equivalent equation will result if

1. The same quantity is added to or subtracted from each side of a given equation.
2. Each side of a given equation is multiplied by or divided by the same nonzero quantity.

Several examples should remind you of the process of solving equations.

**EXAMPLE 1 Solving a Linear Equation** Solve and check:

$$8x - 3(x - 4) = 3(x - 4) + 6$$

**SOLUTION**

$$8x - 3(x - 4) = 3(x - 4) + 6$$ Remove parentheses.

$$8x - 3x + 12 = 3x - 12 + 6$$ Combine like terms.

$$5x + 12 = 3x - 6$$ Subtract 3x from both sides.

$$2x = -18$$

$$x = -9$$ Divide both sides by 2.
CHAPTER 1  Linear Equations and Graphs

MATCHED PROBLEM 1  Solve and check: 3x - 2(2x - 5) = 2(x + 3) - 8

Explore & Discuss 1  According to equality property 2, multiplying both sides of an equation by a nonzero number always produces an equivalent equation. What is the smallest positive number that you could use to multiply both sides of the following equation to produce an equivalent equation without fractions?

\[
\frac{x + 1}{3} - \frac{x}{4} = \frac{1}{2}
\]

EXAMPLE 2  Solving a Linear Equation  Solve and check: \( \frac{x + 2}{2} - \frac{x}{3} = 5 \)

SOLUTION  What operations can we perform on

\[
\frac{x + 2}{2} - \frac{x}{3} = 5
\]

to eliminate the denominators? If we can find a number that is exactly divisible by each denominator, we can use the multiplication property of equality to clear the denominators. The LCD (least common denominator) of the fractions, 6, is exactly what we are looking for! Actually, any common denominator will do, but the LCD results in a simpler equivalent equation. Thus, we multiply both sides of the equation by 6:

\[
6 \left( \frac{x + 2}{2} - \frac{x}{3} \right) = 6 \cdot 5
\]

Remove parentheses.

Combine like terms.

Subtract 6 from both sides.

\[
x + 6 - 2x = 30
\]

\[
x = 24
\]

CHECK  \( \frac{x + 2}{2} - \frac{x}{3} = 5 \)

\[
\frac{24 + 2}{2} - \frac{24}{3} = 5
\]

\[
13 - 8 = 5
\]

\[
5 = 5
\]

MATCHED PROBLEM 2  Solve and check: \( \frac{x + 1}{3} - \frac{x}{4} = \frac{1}{2} \)

In many applications of algebra, formulas or equations must be changed to alternative equivalent forms. The following examples are typical.

* Dashed boxes are used throughout the book to denote steps that are usually performed mentally.
Section 1-1 Linear Equations and Inequalities

Linear Inequalities

Before we start solving linear inequalities, let us recall what we mean by (less than) and (greater than). If \(a\) and \(b\) are real numbers, we write

\[ a \text{ is less than } b. \]

if there exists a positive number \(p\) such that Certainly, we would expect that if a positive number was added to any real number, the sum would be larger than the original. That is essentially what the definition states. If we may also write

\[ b \text{ is greater than } a. \]

\[ b > a \]

Solving a Formula for a Particular Variable

Solve the amount formula for simple interest, \(A = P + Prt\), for

(A) \(r\) in terms of the other variables
(B) \(P\) in terms of the other variables

**SOLUTION**

(A) \[ A = P + Prt \]

Reverse equation.

\[ P + Prt = A \]

Now isolate \(r\) on the left side.

\[ Prt = A - P \]

Divide both members by \(Pr\).

\[ r = \frac{A - P}{Pr} \]

(B) \[ A = P + Prt \]

Reverse equation.

\[ P + Prt = A \]

Factor out \(P\) (note the use of the distributive property).

\[ P(1 + rt) = A \]

Divide by \((1 + rt)\).

\[ P = \frac{A}{1 + rt} \]

**Matched Problem 3** Solve \(M = Nt + Nr\) for

(A) \(t\) (B) \(N\)

Linear Inequalities

Before we start solving linear inequalities, let us recall what we mean by \(<\) (less than) and \(>\) (greater than). If \(a\) and \(b\) are real numbers, we write

\[ a < b \]

\(a\) is less than \(b\).

if there exists a positive number \(p\) such that \(a + p = b\). Certainly, we would expect that if a positive number was added to any real number, the sum would be larger than the original. That is essentially what the definition states. If \(a < b\), we may also write

\[ b > a \]

\(b\) is greater than \(a\).

**Example 4**

**Inequalities**

(A) \(3 < 5\) \quad \text{Since} \ 3 + 2 = 5

(B) \(-6 < -2\) \quad \text{Since} \ -6 + 4 = -2

(C) \(0 > -10\) \quad \text{Since} \ -10 < 0

**Matched Problem 4** Replace each question mark with either \(<\) or \(>\).

(A) \(2 ? 8\) \quad (B) \(-20 ? 0\) \quad (C) \(-3 ? -30\)

The inequality symbols have a very clear geometric interpretation on the real number line. If \(a < b\), then \(a\) is to the left of \(b\) on the number line; if \(c > d\), then \(c\) is to the right of \(d\) (Fig. 1). Check this geometric property with the inequalities in Example 4.

**Explore & Discuss 2** Replace \(?\) with \(<\) or \(>\) in each of the following:

(A) \(-1 ? 3\) and \(2(-1) ? 2(3)\)

(B) \(-1 ? 3\) and \(-2(-1) ? -2(3)\)
Based on these examples, describe verbally the effect of multiplying both sides of an inequality by a number.

Now let us turn to the problem of solving linear inequalities in one variable. Recall that a solution of an inequality involving one variable is a number that, when substituted for the variable, makes the inequality true. The set of all solutions is called the solution set. When we say that we solve an inequality, we mean that we find its solution set. The procedures used to solve linear inequalities in one variable are almost the same as those used to solve linear equations in one variable but with one important exception, as noted in property 3 below.

**THEOREM 2**

**INEQUALITY PROPERTIES**

An equivalent inequality will result and the sense or direction will remain the same if each side of the original inequality

1. Has the same real number added to or subtracted from it.
2. Is multiplied or divided by the same positive number.

An equivalent inequality will result and the sense or direction will reverse if each side of the original inequality:

3. Is multiplied or divided by the same negative number.

Note: Multiplication by 0 and division by 0 are not permitted.

Thus, we can perform essentially the same operations on inequalities that we perform on equations, with the exception that the sense of the inequality reverses if we multiply or divide both sides by a negative number. Otherwise, the sense of the inequality does not change. For example, if we start with the true statement

\[-3 > -7\]

and multiply both sides by 2, we obtain

\[-6 > -14\]

and the sense of the inequality stays the same. But if we multiply both sides of \(-3 > -7\) by \(-2\), the left side becomes 6 and the right side becomes 14, so we must write

\[6 < 14\]

to have a true statement. Thus, the sense of the inequality reverses.

If \(a < b\), the **double inequality** \(a < x < b\) means that \(a < x\) and \(x < b\); that is, \(x\) is between \(a\) and \(b\). **Interval notation** is also used to describe sets defined by inequalities, as shown in Table 1.

The numbers \(a\) and \(b\) in Table 1 are called the **endpoints** of the interval. An interval is **closed** if it contains all its endpoints and **open** if it does not contain any of its endpoints. Thus, the intervals \([a, b]\), \((\infty, a]\), and \([b, \infty)\) are closed and the intervals \((a, b)\), \((\infty, a)\), and \((b, \infty)\) are open. Note that the symbol \(\infty\) (read infinity) is not a number. When we write \([b, \infty)\) we are simply referring to the interval that starts at \(b\) and continues indefinitely to the right. We never refer to \(\infty\) as an endpoint and we never write \([b, \infty]\). The interval \((\infty, \infty)\) is the entire real number line.

Note that an endpoint of a line graph in Table 1 has a square bracket through it if the endpoint is included in the interval and a parenthesis through it if it is not.
The notation (2, 7) has two common mathematical interpretations: the ordered pair with first coordinate 2 and second coordinate 7, and the open interval consisting of all real numbers between 2 and 7. The choice of interpretations is usually determined by the context in which the notation is used. The notation (2, -7) also can be interpreted as an ordered pair, but not as an interval. In interval notation, it is always assumed that the left endpoint is less than the right endpoint. Thus, (2, 7) is correct interval notation, but (2, -7) is not.

**Table 1**

<table>
<thead>
<tr>
<th>Interval Notation</th>
<th>Inequality Notation</th>
<th>Line Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>[a, b]</td>
<td>( a \leq x \leq b )</td>
<td>![Graph of [a, b]]</td>
</tr>
<tr>
<td>(a, b)</td>
<td>( a \leq x &lt; b )</td>
<td>![Graph of (a, b)]</td>
</tr>
<tr>
<td>(a, b)</td>
<td>( a &lt; x \leq b )</td>
<td>![Graph of (a, b)]</td>
</tr>
<tr>
<td>(a, b)</td>
<td>( a &lt; x &lt; b )</td>
<td>![Graph of (a, b)]</td>
</tr>
<tr>
<td>(-(\infty), a)</td>
<td>( x \leq a )</td>
<td>![Graph of (-(\infty), a)]</td>
</tr>
<tr>
<td>(-(\infty), a)</td>
<td>( x &lt; a )</td>
<td>![Graph of (-(\infty), a)]</td>
</tr>
<tr>
<td>(b, (\infty))</td>
<td>( x \geq b )</td>
<td>![Graph of (b, (\infty))]</td>
</tr>
<tr>
<td>(b, (\infty))</td>
<td>( x &gt; b )</td>
<td>![Graph of (b, (\infty))]</td>
</tr>
</tbody>
</table>

**Example 5**

Interval and Inequality Notation, and Line Graphs

(A) Write \([-2, 3]\) as a double inequality and graph.
(B) Write \(x \geq -5\) in interval notation and graph.

**Solution**

(A) \([-2, 3]\) is equivalent to \(-2 \leq x < 3\).
(B) \(x \geq -5\) is equivalent to \([-5, \infty)\).

**Matched Problem 5**

(A) Write \((-7, 4]\) as a double inequality and graph.
(B) Write \(x < 3\) in interval notation and graph.

**Explore & Discuss 3**

The solution to Example 5B shows the graph of the inequality \(x \geq -5\). What is the graph of \(x < -5\)? What is the corresponding interval? Describe the relationship between these sets.
APPLICATIONS

To realize the full potential of algebra, we must be able to translate real-world problems into mathematical forms. In short, we must be able to do word problems. There are many different types of applications—so many, in fact, that no single approach applies to all. However, the following suggestions may help you get started:

Procedure for Solving Word Problems

1. Read the problem carefully and introduce a variable to represent an unknown quantity in the problem. Often the question asked in a problem will indicate the best way to introduce this variable.

Solving a Linear Inequality

Solve and graph:

\[ 2(2x + 3) < 6(x - 2) + 10 \]

**SOLUTION**

\[
\begin{align*}
2(2x + 3) & < 6(x - 2) + 10 \\
4x + 6 & < 6x - 12 + 10 \\
4x + 6 & < 6x - 2 \\
-2x + 6 & < -2 \\
-2x & < -8 \\
x & > 4 \quad \text{or} \quad (4, \infty)
\end{align*}
\]

Notice that in the graph of \( x > 4 \), we use a parenthesis through 4, since the point 4 is not included in the graph.

**MATCHED PROBLEM 6**

Solve and graph: \( 3(x - 1) \leq 5(x + 2) - 5 \)

Solving a Double Inequality

Solve and graph: \(-3 < 2x + 3 \leq 9\)

**SOLUTION**

We are looking for all numbers \( x \) such that \( 2x + 3 \) is between \(-3\) and \(9\), including \(9\) but not \(-3\). We proceed as before except that we try to isolate \( x \) in the middle:

\[
\begin{align*}
-3 & < 2x + 3 \leq 9 \\
\frac{-3}{2} & < x \leq \frac{3}{2} \\
-3 & < x \leq 3 \quad \text{or} \quad (-3, 3]
\end{align*}
\]

**MATCHED PROBLEM 7**

Solve and graph: \( -8 \leq 3x - 5 < 7 \)

Note that a linear equation usually has exactly one solution, while a linear inequality usually has infinitely many solutions.

APPLICATIONS

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**Procedure for Solving Word Problems**

1. Read the problem carefully and introduce a variable to represent an unknown quantity in the problem. Often the question asked in a problem will indicate the best way to introduce this variable.
2. Identify other quantities in the problem (known or unknown) and, whenever possible, express unknown quantities in terms of the variable you introduced in step 1.

3. Write a verbal statement using the conditions stated in the problem and then write an equivalent mathematical statement (equation or inequality).

4. Solve the equation or inequality and answer the questions posed in the problem.

5. Check the solution(s) in the original problem.

**Example 8**

**Purchase Price**

John purchases a computer from an online store for $851.26, including a $57 shipping charge and 5.2% state sales tax. What is the purchase price of the computer?

**SOLUTION**

**Step 1.** After reading the problem, we decide to let \( x \) represent the purchase price of the computer.

**Step 2.** Identify quantities in the problem.

- Shipping charges: $57
- Sales tax: 0.052\( x \)
- Total cost: $851.26

**Step 3.** Write a verbal statement and an equation.

\[
\text{Price} + \text{Shipping Charges} + \text{Sales Tax} = \text{Total Order Cost}
\]

\[
x + 57 + 0.052x = 851.26
\]

**Step 4.** Solve the equation and answer the question.

\[
x + 57 + 0.052x = 851.26 \quad \text{Collect like terms.}
\]

\[
1.052x + 57 = 851.26 \quad \text{Subtract 57 from both sides.}
\]

\[
1.052x = 794.26 \quad \text{Divide both sides by 1.052.}
\]

\[
x = 755
\]

The price of the computer is $755.

**Step 5.** Check the answer in the original problem.

\[
\begin{align*}
\text{Price} &= 755.00 \\
\text{Shipping charges} &= 57.00 \\
\text{Tax} \ 0.052 \cdot 755 &= 39.26 \\
\text{Total} &= 851.26
\end{align*}
\]

**Matched Problem 8**

Mary paid 8.5% sales tax and a $190 title and license fee when she bought a new car for a total of $28,400. What is the purchase price of the car?

**Example 9**

**Break-Even Analysis**

A recording company produces compact disks (CDs). One-time fixed costs for a particular CD are $24,000, which includes costs such as recording, album design, and promotion. Variable costs amount to $6.20 per CD and include the manufacturing, distribution, and royalty costs for each disk actually manufactured and sold to a retailer. The CD is sold to retail outlets at $8.70 each. How many CDs must be manufactured and sold for the company to break even?
CHAPTER 1 Linear Equations and Graphs

**SOLUTION**

**Step 1.** Let \( x \) = number of CDs manufactured and sold.

**Step 2.**

- \( C = \text{cost of producing } x \text{ CDs} \)
- \( R = \text{revenue (return) on sales of } x \text{ CDs} \)
- Fixed costs = $24,000
- Variable costs = $6.20x

\[ C = \text{Fixed costs} + \text{variable costs} = 24,000 + 6.20x \]

\[ R = 8.70x \]

**Step 3.** The company breaks even if \( R = C \); that is, if

\[ 8.70x = 24,000 + 6.20x \]

**Step 4.**

- Subtract 6.2x from both sides.
- Divide both sides by 2.5.

\[ 2.5x = 24,000 \]
\[ x = 9,600 \]

The company must make and sell 9,600 CDs to break even.

**Step 5.** Check:

\[
\begin{array}{c|c|c}
\text{Costs} & \text{Revenue} \\
24,000 & 8.7(9,600) \\
24,000 + 6.2(9,600) & 83,520
\end{array}
\]

**MATCHED PROBLEM 9**

How many CDs would a recording company have to make and sell to break even if the fixed costs are $18,000, variable costs are $5.20 per CD, and the CDs are sold to retailers for $7.60 each?

**EXAMPLE 10**

**Consumer Price Index**

The Consumer Price Index (CPI) is a measure of the average change in prices over time from a designated reference period, which equals 100. The index is based on prices of basic consumer goods and services, and is published at regular intervals by the Bureau of Labor Statistics. Table 2 lists the CPI for several years from 1960 to 2000. What net annual salary in 2000 would have the same purchasing power as a net annual salary of $13,000 in 1960? Compute the answer to the nearest dollar.

**SOLUTION**

**Step 1.** Let \( x \) = the purchasing power of an annual salary in 2000.

**Step 2.** Annual salary in 1960 = $13,000

\[
\begin{align*}
\text{CPI in 1960} &= 29.6 \\
\text{CPI in 2000} &= 172.2
\end{align*}
\]

**Step 3.** The ratio of a salary in 2000 to a salary in 1960 is the same as the ratio of the CPI in 2000 to the CPI in 1960.

\[
\frac{x}{13,000} = \frac{172.2}{29.6}
\]

Multiply both sides by 13,000.

**Step 4.**

\[
x = 13,000 \times \frac{172.2}{29.6} = 75,628 \text{ per year}
\]

**Step 5.**

\[
\begin{array}{c|c|c}
\text{Salary Ratio} & \text{CPI Ratio} \\
\frac{75,628}{13,000} & \frac{172.2}{29.6} = 5.81757
\end{array}
\]

[Note: The slight difference in these ratios is due to rounding the 2000 salary to the nearest dollar.]
What net annual salary in 1970 would have had the same purchasing power as a net annual salary of $100,000 in 2000? Compute the answer to the nearest dollar.

Answers to Matched Problems

1. $x = 4$
2. $x = 2$
3. (A) \( t = \frac{M - Nr}{N} \)
   (B) \( N = \frac{M}{t + r} \)
4. (A) < (B) < (C) >
5. (A) \(-7 < x \leq 4\)
   (B) \((-\infty, 3] \)
6. \( x \approx -4 \text{ or } [-4, \infty) \)
7. \(-1 \leq x < 4 \text{ or } [-1, 4) \)
8. $26,000$

9. 7,500 CDs
10. $22,532$

Exercise 1-1

A Solve Problems 1–6.
1. \(2m + 9 = 5m - 6\)
2. \(3y - 4 = 6y - 19\)
3. \(2x + 3 < -4\)
4. \(5x + 2 > 1\)
5. \(-3x \approx -12\)
6. \(-4x \approx 8\)

Solve Problems 7–10 and graph.
7. \(-4x - 7 > 5\)
8. \(-2x + 8 < 4\)
9. \(2 \leq x + 3 \leq 5\)
10. \(-4 < 2y - 3 < 9\)

11. \(\frac{x}{3} - \frac{1}{2} = \frac{1}{3}\)
12. \(\frac{m}{5} - 2 = \frac{3}{5}\)
13. \(\frac{x}{3} \geq \frac{-5}{4}\)
14. \(\frac{y}{4} \leq -1\)
15. \(\frac{y}{3} = 4 - \frac{y}{6}\)
16. \(\frac{x}{4} = 9 - \frac{x}{2}\)

B
17. \(10x + 25(x - 3) = 275\)
18. \(-3(4 - x) = 5 - (x + 1)\)
19. \(3 - y \leq 4(y - 3)\)
20. \(x - 2 \approx 2(x - 5)\)
21. \(\frac{x}{5} - \frac{x}{6} = \frac{6}{5}\)
22. \(\frac{y}{4} - \frac{y}{3} = \frac{1}{2}\)
23. \(\frac{m}{3} - 3 < \frac{3}{5} - \frac{m}{2}\)
24. \(\frac{u}{2} - \frac{2}{3} < \frac{u}{3} + 2\)
25. \(0.1(x - 7) + 0.05x = 0.8\)
26. \(0.03(2x + 1) - 0.05x = 12\)

Solve Problems 27–30 and graph.
27. \(2 \leq 3x - 7 < 14\)
28. \(-4 < 5x + 6 < 21\)
29. \(-4 \leq \frac{2}{5}C + 32 \leq 68\)
30. \(-1 \leq \frac{2}{3}x + 5 \leq 11\)

C Solve Problems 31–38 for the indicated variable.
31. \(3x - 4y = 12; \text{ for } y\)
32. \(y = -\frac{2}{3}x + 8; \text{ for } x\)
33. \(Ax + By = C; \text{ for } y (B \neq 0)\)
34. \(y = mx + b; \text{ for } m\)
35. \(F = \frac{2}{3}C + 32; \text{ for } C\)
36. \(C = \frac{3}{5}(F - 32); \text{ for } F\)
37. \(A = \frac{3}{5}(Bm - Bn); \text{ for } B\)
38. \(X = \frac{1}{3}(3CD - C); \text{ for } C\)

Solve Problems 39 and 40 and graph.
39. \(-3 \leq 4 - 7x < 18\)
40. \(-10 \leq 8 - 3u \leq -6\)

41. What can be said about the signs of the numbers \(a\) and \(b\) in each case?
   (A) \(ab > 0\)
   (B) \(ab < 0\)
   (C) \(\frac{a}{b} > 0\)
   (D) \(\frac{a}{b} < 0\)

42. What can be said about the signs of the numbers \(a, b,\) and \(c\) in each case?
   (A) \(abc > 0\)
   (B) \(\frac{ab}{c} < 0\)
   (C) \(\frac{a}{bc} > 0\)
   (D) \(\frac{a^2}{bc} < 0\)
43. Replace each question mark with < or >, as appropriate:
   (A) If \( a - b = 2 \), then \( a ? b \).
   (B) If \( c - d = -1 \), then \( c ? d \).

44. For what \( c \) and \( d \) is \( c + d < c - d \)?

45. If both \( a \) and \( b \) are positive numbers and \( b/a \) is greater than 1, then is \( a - b \) positive or negative?

46. If both \( a \) and \( b \) are negative numbers and \( b/a \) is greater than 1, then is \( a - b \) positive or negative?

In Problems 47–52, discuss the validity of each statement. If the statement is true, explain why. If not, give a counterexample.

47. If the intersection of two open intervals is nonempty, then their intersection is an open interval.

48. If the intersection of two closed intervals is nonempty, then their intersection is a closed interval.

49. The union of any two open intervals is an open interval.

50. The union of any two closed intervals is a closed interval.

51. If the intersection of two open intervals is nonempty, then their union is an open interval.

52. If the intersection of two closed intervals is nonempty, then their union is a closed interval.

45. Inflation. Retail and wholesale prices. IRA. Equipment rental.

46. Retail and sale prices. IRA. Equipment rental.

53. **Puzzle.** A jazz concert bought in $165,000 on the sale of 8,000 tickets. If the tickets sold for $15 and $25 each, how many of each type of ticket were sold?

54. **Puzzle.** An all-day parking meter takes only dimes and quarters. If it contains 100 coins with a total value of $14.50, how many of each type of coin are in the meter?

55. **IRA.** You have $500,000 in an IRA (Individual Retirement Account) at the time you retire. You have the option of investing this money in two funds: Fund A pays 5.2% annually and Fund B pays 7.7% annually. How much would you divide your money between Fund A and Fund B to produce an annual interest income of $34,000?

56. **IRA.** Refer to Problem 55. How should you divide your money between Fund A and Fund B to produce an annual interest income of $30,000?

57. **Inflation.** If the price change of cars parallels the change in the CPI (see Table 2 in Example 10), what would a car sell for (to the nearest dollar) in 2000 if a comparable model sold for $5,000 in 1970?

58. **Inflation.** If the price change in houses parallels the CPI (see Table 2 in Example 10), what would a house valued at $200,000 in 2000 be valued at (to the nearest dollar) in 1960?

59. **Retail and wholesale prices.** Retail prices in a department store are obtained by marking up the wholesale price by 40%. That is, retail price is obtained by adding 40% of the wholesale price to the wholesale price.
   (A) What is the retail price of a suit if the wholesale price is $300?
   (B) What is the wholesale price of a pair of jeans if the retail price is $77?

60. **Retail and sale prices.** Sale prices in a department store are obtained by marking down the retail price by 15%. That is, sale price is obtained by subtracting 15% of the retail price from the retail price.
   (A) What is the sale price of a hat that has a retail price of $60?
   (B) What is the retail price of a dress that has a sales price of $68?

61. **Equipment rental.** A golf course charges $52 for a round of golf using a set of their clubs and $44 if you have your own clubs. How many rounds must you play with your clubs to recover the cost of the clubs?

62. **Equipment rental.** The local supermarket rents carpet cleaners for $20 a day. These cleaners use shampoo in a special cartridge that sells for $16 and is available only from the supermarket. A home carpet cleaner can be purchased for $300. Shampoo for the home cleaner is readily available for $9 a bottle. Past experience has shown that it takes two shampoo cartridges to clean the 10-foot-by-12-foot carpet in your living room with the rented cleaner. Cleaning the same area with the home cleaner will consume three bottles of shampoo. If you buy the home cleaner, how many times must you clean the living room carpet to make buying cheaper than renting?

63. **Sales commissions.** One employee of a computer store is paid a base salary of $2,000 a month plus an 8% commission on all sales during the month. How much must the employee sell in one month to earn a monthly income of $4,000?

64. **Sales commissions.** A second employee of the computer store in Problem 63 is paid a base salary of $3,000 a month plus a 5% commission on all sales during the month.
   (A) How much must this employee sell in one month to earn a total of $4,000 for the month?
   (B) Determine the sales level at which both employees receive the same monthly income.
   (C) If employees can select either of these payment methods, how would you advise an employee to make this selection?

65. **Break-even analysis.** A publisher for a promising new novel figures fixed costs (overhead, advances, promotion, copy editing, typesetting, and so on) at $55,000, and variable costs (printing, paper, binding, shipping) at $1.60 for each book produced. If the book is sold to distributors for $11 each, how many must be produced and sold for the publisher to break even?
66. **Break-even analysis.** The publisher of a new book called *Muscle-Powered Sports* figures fixed costs at $92,000 and variable costs at $2.10 for each book produced. If the book is sold to distributors for $15 each, how many must be sold for the publisher to break even?

67. **Break-even analysis.** The publisher in Problem 66 finds that rising prices for paper increase the variable costs to $2.10 per book.
   (A) Discuss possible strategies the company might use to deal with this increase in costs.
   (B) If the company continues to sell the books for $15, how many books must they sell now to make a profit?
   (C) If the company wants to start making a profit at the same production level as before the cost increase, how much should they sell the book for now?

68. **Break-even analysis.** The publisher in Problem 66 finds that rising prices for paper increase the variable costs to $2.70 per book.
   (A) Discuss possible strategies the company might use to deal with this increase in costs.
   (B) If the company continues to sell the books for $15, how many books must they sell now to make a profit?
   (C) If the company wants to start making a profit at the same production level as before the cost increase, how much should they sell the book for now?

69. **Wildlife management.** A naturalist for a fish and game department estimated the total number of rainbow trout in a certain lake using the popular capture–mark–recapture technique. He netted, marked, and released 200 rainbow trout. A week later, allowing for thorough mixing, he again netted 200 trout and found 8 marked ones among them. Assuming that the proportion of marked fish in the second sample was the same as the proportion of all marked fish in the total population, estimate the number of rainbow trout in the lake.

70. **Ecology.** If the temperature for a 24 hour period at an Antarctic station ranged between −49°F and 14°F (that is, −49 ≤ F ≤ 14), what was the range in degrees Celsius? [Note: F = \(\frac{5}{9}C + 32\).]

71. **Psychology.** The IQ (intelligence quotient) is found by dividing the mental age (MA), as indicated on standard tests, by the chronological age (CA) and multiplying by 100. For example, if a child has a mental age of 12 and a chronological age of 8, the calculated IQ is 150. If a 9-year-old girl has an IQ of 140, compute her mental age.

72. **Psychology.** Refer to Problem 71. If the IQ of a group of 12-year-old children varies between 80 and 140, what is the range of their mental ages?

73. **Anthropology.** In their study of genetic groupings, anthropologists use a ratio called the cephalic index. This is the ratio of the breadth \(B\) of the head to its length \(L\) (looking down from above) expressed as a percentage. A study of the Gurung community of Nepal published in the *Kathmandu University Medical Journal* in 2005 found that the average head length of males was 18 cm and their head breadths varied between 12 and 18 cm. Find the range of the cephalic index for males. Round endpoints to one decimal place.

74. **Anthropology.** Refer to Problem 73. The same study found that the average head length of females was 17.4 cm and their head breadths varied between 15 and 20 cm. Find the range of the cephalic index for females. Round endpoints to one decimal place.

---

**Section 1-2  GRAPHS AND LINES**

- Cartesian Coordinate System
- Graphs of \(Ax + By = C\)
- Slope of a Line
- Equations of Lines: Special Forms
- Applications

In this section we will consider one of the most basic geometric figures—a line. When we use the term line in this book, we mean _straight line_. We will learn how to recognize and graph a line, and how to use information concerning a line to find its equation. Examining the graph of any equation often results in additional insight into the nature of the equation’s solutions.
CHAPTER 1  Linear Equations and Graphs

- Cartesian Coordinate System

Recall that to form a Cartesian or rectangular coordinate system, we select two real number lines, one horizontal and one vertical, and let them cross through their origins as indicated in Figure 1. Up and to the right are the usual choices for the positive directions. These two number lines are called the horizontal axis and the vertical axis, or, together, the coordinate axes. The horizontal axis is usually referred to as the x axis and the vertical axis as the y axis, and each is labeled accordingly. Other labels may be used in certain situations. The coordinate axes divide the plane into four parts called quadrants, which are numbered counterclockwise from I to IV (see Fig. 1).

![Cartesian Coordinate System](image)

**FIGURE 1** The Cartesian (rectangular) coordinate system

Now we want to assign coordinates to each point in the plane. Given an arbitrary point $P$ in the plane, pass horizontal and vertical lines through the point (Fig. 1). The vertical line will intersect the horizontal axis at a point with coordinate $a$, and the horizontal line will intersect the vertical axis at a point with coordinate $b$. These two numbers written as the ordered pair* $(a, b)$ form the coordinates of the point $P$. The first coordinate, $a$, is called the abscissa of $P$; the second coordinate, $b$, is called the ordinate of $P$. The abscissa of $Q$ in Figure 1 is $-5$, and the ordinate of $Q$ is $5$. The coordinates of a point can also be referenced in terms of the axis labels. The $x$ coordinate of $R$ in Figure 1 is $10$, and the $y$ coordinate of $R$ is $-10$. The point with coordinates $(0, 0)$ is called the origin.

The procedure we have just described assigns to each point $P$ in the plane a unique pair of real numbers $(a, b)$. Conversely, if we are given an ordered pair of real numbers $(a, b)$, then, reversing this procedure, we can determine a unique point $P$ in the plane. Thus,

*There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.*

This is often referred to as the fundamental theorem of analytic geometry.

- Graphs of $Ax + By = C$

In Section 1-1, we called an equation of the form $ax + b = 0$ ($a \neq 0$) a linear equation in one variable. Now we want to consider linear equations in two variables:

---

*An ordered pair of real numbers is a pair of numbers in which the order is specified. We now use $(a, b)$ as the coordinates of a point in a plane. In Section 1-1, we used $(a, b)$ to represent an interval on a real number line. These concepts are not the same. You must always interpret the symbol $(a, b)$ in terms of the context in which it is used.*
A linear equation in two variables is an equation that can be written in the standard form

$$Ax + By = C$$

where $A$, $B$, and $C$ are constants ($A$ and $B$ not both 0) and $x$ and $y$ are variables.

A solution of an equation in two variables is an ordered pair of real numbers that satisfy the equation. For example, $(4, 3)$ is a solution of $3x - 2y = 6$. The solution set of an equation in two variables is the set of all solutions of the equation. The graph of an equation is the graph of its solution set.

**Explore & Discuss 1**

(A) As noted earlier, $(4, 3)$ is a solution of the equation

$$3x - 2y = 6$$

Find three more solutions of this equation. Plot these solutions in a Cartesian coordinate system. What familiar geometric shape could be used to describe the solution set of this equation?

(B) Repeat part (A) for the equation $x = 2$.

(C) Repeat part (A) for the equation $y = -3$.

In Explore–Discuss 1, you should have recognized that the graph of each equation is a (straight) line. Theorem 1 confirms this fact.

**THEOREM 1**

**GRAPH OF A LINEAR EQUATION IN TWO VARIABLES**

The graph of any equation of the form

$$Ax + By = C \quad (A \text{ and } B \text{ not both 0})$$

is a line, and any line in a Cartesian coordinate system is the graph of an equation of this form.

If $A \neq 0$ and $B \neq 0$, then equation (1) can be written as

$$y = \frac{-A}{B}x + \frac{C}{B} = mx + b, m = 0$$

a form we will use often. If $A = 0$ and $B \neq 0$, then equation (1) can be written as

$$y = \frac{C}{B}$$

and its graph is a horizontal line. If $A \neq 0$ and $B = 0$, then equation (1) can be written as

$$x = \frac{A}{B}$$

and its graph is a vertical line. To graph equation (1), or any of its special cases, plot any two points in the solution set and use a straightedge to draw the line through these two points. The points where the line crosses the axes are often the easiest to find. The $y$ intercept* is the $y$ coordinate of the point where the graph crosses the $y$ axis and the $x$ intercept is the $x$ coordinate of the point where the graph crosses the $x$ axis. To find the $y$ intercept, let $x = 0$ and solve for $y$. To find the $x$ intercept, let $y = 0$ and solve for $x$. It is a good idea to find a third point as a check point.

* If the $x$ intercept is $a$ and the $y$ intercept is $b$, then the graph of the line passes through the points $(a, 0)$ and $(0, b)$. It is common practice to refer to both the numbers $a$ and $b$ and the points $(a, 0)$ and $(0, b)$ as the $x$ and $y$ intercepts of the line.
**EXAMPLE 1** Using Intercepts to Graph a Line

Graph: \(3x - 4y = 12\)

**SOLUTION**

\[
\begin{array}{c|c|c}
 x & y & \\
0 & -3 & \text{y intercept} \\
4 & 0 & \text{x intercept} \\
8 & 3 & \text{Check point} \\
\end{array}
\]

**MATCHED PROBLEM 1** Graph: \(4x - 3y = 12\)

The icon in the margin is used throughout this book to identify optional graphing calculator activities that are intended to give you additional insight into the concepts under discussion. You may have to consult the manual for your calculator* for the details necessary to carry out these activities.

**EXAMPLE 2** Using a Graphing Calculator

Graph \(3x - 4y = 12\) on a graphing calculator and find the intercepts.

**SOLUTION**

First, we solve \(3x - 4y = 12\) for \(y\).

\[
\begin{align*}
3x - 4y &= 12 \\
-4y &= -3x + 12 \\
y &= \frac{-3x + 12}{-4} \\
y &= \frac{3}{4}x - 3
\end{align*}
\]

Now we enter the right side of equation (2) in a calculator (Fig. 2A), enter values for the window variables (Fig. 2B), and graph the line (Fig. 2C).

**FIGURE 2** Graphing a line on a graphing calculator

Next we use two calculator routines to find the intercepts: **TRACE** (Fig. 3A) and **zero** (Fig. 3B).

* We used a Texas Instruments graphing calculator from the TI-83/84 family to produce the graphing calculator screens in the book. Manuals for most graphing calculators are readily available on the Internet.
Section 1-2  Graphs and Lines

Graphs and Lines

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Slope of a Line

If we take two points and on a line, then the ratio of the change in to the change in as the point moves from point to point is called the slope of the line. In a sense, slope provides a measure of the “steepness” of a line relative to the x-axis. The change in is often called the run and the change in the rise.

Definition  Slope of a Line  If a line passes through two distinct points , then its slope is given by the formula

The change in is often called the run and the change in the rise.
For a horizontal line, $y$ does not change; hence, its slope is 0. For a vertical line, $x$ does not change; hence, $x_1 = x_2$ and its slope is not defined. In general, the slope of a line may be positive, negative, 0, or not defined. Each case is illustrated geometrically in Table 1.

**TABLE 1 Geometric Interpretation of Slope**

<table>
<thead>
<tr>
<th>Line</th>
<th>Rising as $x$ moves from left to right</th>
<th>Falling as $x$ moves from left to right</th>
<th>Horizontal</th>
<th>Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>Positive</td>
<td>Negative</td>
<td>0</td>
<td>Not defined</td>
</tr>
<tr>
<td>Example</td>
<td>![Image of rising line]</td>
<td>![Image of falling line]</td>
<td>![Image of horizontal line]</td>
<td>![Image of vertical line]</td>
</tr>
</tbody>
</table>

**INSIGHT**

One property of real numbers discussed in Appendix A, Section A-1, is

$$\frac{-a}{b} = \frac{-a}{b} = \frac{-a}{b} = \frac{a}{b}, \quad b \neq 0$$

This property implies that it does not matter which point we label as $P_1$ and which we label as $P_2$ in the slope formula. For example, if $A = (4, 3)$ and $B = (1, 2)$, then

$$B = P_2 = (1, 2) \quad A = P_2 = (4, 3)$$
$$A = P_1 = (4, 3) \quad B = P_1 = (1, 2)$$

$$m = \frac{2 - 3}{1 - 4} = \frac{-1}{-3} = \frac{1}{3} = \frac{3 - 2}{4 - 1}$$

A property of similar triangles (see Table II in Appendix C) ensures that the slope of a line is the same for any pair of distinct points on the line (Fig. 4).

**EXAMPLE 4 Finding Slopes** Sketch a line through each pair of points, and find the slope of each line.

(A) $(-3, -2), (3, 4)$  
(B) $(-1, 3), (2, -3)$

(C) $(-2, -3), (3, -3)$  
(D) $(-2, 4), (-2, -2)$
SOLUTION

(A) \[ m = \frac{4 - (-2)}{3 - (-3)} = \frac{6}{6} = 1 \]

(B) \[ m = \frac{-3 - 3}{2 - (-1)} = \frac{-6}{3} = -2 \]

(C) \[ m = \frac{-3 - (-3)}{3 - (-2)} = \frac{0}{5} = 0 \]

(D) Slope is not defined.

MATCHED PROBLEM 4
Find the slope of the line through each pair of points.

(A) (-2, 4), (3, 4)  
(B) (-2, 4), (0, -4)  
(C) (-1, 5), (-1, -2)  
(D) (-2, 4), (1, -2), (2, 1)

Equations of Lines: Special Forms
Let us start by investigating why \( y = mx + b \) is called the slope-intercept form for a line.

Explore & Discuss 2

(A) Graph \( y = x + b \) for \( b = -5, -3, 0, 3, \) and \( 5 \) simultaneously in the same coordinate system. Verbally describe the geometric significance of \( b \).

(B) Graph \( y = mx - 1 \) for \( m = -2, -1, 0, 1, \) and \( 2 \) simultaneously in the same coordinate system. Verbally describe the geometric significance of \( m \).

(C) Using a graphing utility, explore the graph of \( y = mx + b \) for different values of \( m \) and \( b \).

As you can see from Explore–Discuss 2, constants \( m \) and \( b \) in \( y = mx + b \) have special geometric significance, which we now explicitly state.

If we let \( x = 0 \), then \( y = b \), and we observe that the graph of \( y = mx + b \) crosses the \( y \) axis at \((0, b)\). The constant \( b \) is the \( y \) intercept. For example, the \( y \) intercept of the graph of \( y = -4x - 1 \) is -1.

To determine the geometric significance of \( m \), we proceed as follows: If \( y = mx + b \), then by setting \( x = 0 \) and \( x = 1 \), we conclude that \((0, b)\) and \((1, m + b)\) lie on its graph (Fig. 5). Hence, the slope of this line is given by:

\[ \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(m + b) - b}{1 - 0} = m \]

Thus, \( m \) is the slope of the line given by \( y = mx + b \).
DEFINITION **Slope-Intercept Form**

The equation

\[ y = mx + b \]

is called the **slope-intercept form** of an equation of a line.

**EXAMPLE 5** Using the Slope-Intercept Form

(A) Find the slope and \( y \) intercept, and graph \( y = -\frac{2}{3}x - 3 \).

(B) Write the equation of the line with slope \( \frac{2}{3} \) and \( y \) intercept \(-2\).

**SOLUTION**

(A) Slope \( m = -\frac{2}{3} \)

\( y \) intercept \( b = -3 \)

(B) \( m = \frac{2}{3} \) and \( b = -2 \);

thus, \( y = \frac{2}{3}x - 2 \)

**MATCHED PROBLEM 5** Write the equation of the line with slope \( \frac{1}{2} \) and \( y \) intercept \(-1\). Graph.

Suppose that a line has slope \( m \) and passes through a fixed point \((x_1, y_1)\). If the point \((x, y)\) is any other point on the line (Fig. 6), then

\[ \frac{y - y_1}{x - x_1} = m \]

That is,

\[ y - y_1 = m(x - x_1) \]

We now observe that \((x_1, y_1)\) also satisfies equation (4) and conclude that equation (4) is an equation of a line with slope \( m \) that passes through \((x_1, y_1)\).

DEFINITION **Point-Slope Form**

An equation of a line with slope \( m \) that passes through \((x_1, y_1)\) is

\[ y - y_1 = m(x - x_1) \]

which is called the **point-slope form** of an equation of a line.

The point-slope form is extremely useful, since it enables us to find an equation for a line if we know its slope and the coordinates of a point on the line or if we know the coordinates of two points on the line.
**Example 6** Using the Point-Slope Form

(A) Find an equation for the line that has slope $\frac{1}{2}$ and passes through $(-4, 3)$. Write the final answer in the form $Ax + By = C$.

(B) Find an equation for the line that passes through the two points $(-3, 2)$ and $(-4, 5)$. Write the resulting equation in the form $y = mx + b$.

**Solution**

(A) Use $y - y_1 = m(x - x_1)$. Let $m = \frac{1}{2}$ and $(x_1, y_1) = (-4, 3)$. Then

\[
y - 3 = \frac{1}{2}[x - (-4)]
\]

\[
y - 3 = \frac{1}{2}(x + 4)
\]

\[
2y - 6 = x + 4
\]

\[
x - 2y = 10 \quad \text{or} \quad x - 2y = -10
\]

(B) First, find the slope of the line by using the slope formula:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{-4 - (-3)} = \frac{3}{-1} = -3
\]

Now use $y - y_1 = m(x - x_1)$ with $m = -3$ and $(x_1, y_1) = (-3, 2)$:

\[
y - 2 = -3[x - (-3)]
\]

\[
y - 2 = -3(x + 3)
\]

\[
y - 2 = -3x - 9
\]

\[
y = -3x - 7
\]

**Matched Problem 6**

(A) Find an equation for the line that has slope $\frac{2}{3}$ and passes through $(6, -2)$. Write the resulting equation in the form $Ax + By = C$, $A > 0$.

(B) Find an equation for the line that passes through $(2, -3)$ and $(4, 3)$. Write the resulting equation in the form $y = mx + b$.

The various forms of the equation of a line that we have discussed are summarized in Table 2 for convenient reference.

<table>
<thead>
<tr>
<th>TABLE 2 Equations of a Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard form</td>
</tr>
<tr>
<td>Slope-intercept form</td>
</tr>
<tr>
<td>Point-slope form</td>
</tr>
<tr>
<td>Horizontal line</td>
</tr>
<tr>
<td>Vertical line</td>
</tr>
</tbody>
</table>

**Applications**

We will now see how equations of lines occur in certain applications.

**Example 7** Cost Equation

The management of a company that manufactures roller skates has fixed costs (costs at 0 output) of $300 per day and total costs of $4,300 per day at an output of 100 pairs of skates per day. Assume that cost $C$ is linearly related to output $x$.

(A) Find the slope of the line joining the points associated with outputs of 0 and 100; that is, the line passing through $(0, 300)$ and $(100, 4,300)$.

(B) Find an equation of the line relating output to cost. Write the final answer in the form $C = mx + b$.

(C) Graph the cost equation from part (B) for $0 \leq x \leq 200$. 
22  Chapter 1  Linear Equations and Graphs

**SOLUTION (A)**

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
\[ = \frac{4,300 - 300}{100 - 0} \]
\[ = \frac{4,000}{100} = 40 \]

(B) We must find an equation of the line that passes through \((0, 300)\) with slope 40. We use the slope-intercept form:

\[ C = mx + b \]
\[ C = 40x + 300 \]

(C) We can interpret the slope 40 as the rate of change of the cost function with respect to production. Since increasing production from \(x\) to \(x + 1\) will increase the cost by $40 (from \(40x + 300\) to \(40x + 340\)), the slope 40 can be interpreted as the rate of change of the cost function with respect to production \(x\).

**MATCHED PROBLEM 7**

Answer parts (A) and (B) in Example 7 for fixed costs of $250 per day and total costs of $3,450 per day at an output of 80 pairs of skates per day.

**EXAMPLE 8**

**Supply and Demand**  Table 3 lists the supply and demand for wheat in the United States during two recent years. Assume that the relationship between supply and price is linear and the relationship between demand and price is also linear.

(A) Find a linear supply equation of the form \( p = mx + b \), where \( p \) is the price per bushel in dollars and \( x \) is the corresponding supply in billions of bushels.

(B) Find a linear demand equation of the form \( p = mx + b \), where \( p \) is the price per bushel in dollars and \( x \) is the corresponding demand in billions of bushels.

(C) Graph the supply and demand equations in the same coordinate system and find their point of intersection.

**TABLE 3 U.S. Wheat Supply and Demand**

<table>
<thead>
<tr>
<th>Year</th>
<th>Supply (mil bu)</th>
<th>Demand (mil bu)</th>
<th>Price ($/bu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>2,480</td>
<td>2,300</td>
<td>3.38</td>
</tr>
<tr>
<td>1999</td>
<td>2,300</td>
<td>2,390</td>
<td>2.48</td>
</tr>
</tbody>
</table>

Source: The ProExporter Network®
SOLUTION

(A) To find a supply equation in the form \( p = mx + b \), we must first find two points of the form \((x, p)\) that supply line passes through. From Table 3, \((2,480, 3.38)\) and \((2,300, 2.48)\) are two such points. First, find the slope of the line:

\[
m = \frac{3.38 - 2.48}{2,480 - 2,300} = \frac{0.9}{180} = 0.005
\]

Now, use the point-slope form to find the equation of the line:

\[
\begin{align*}
p - p_1 &= m(x - x_1) \\
p - 2.48 &= 0.005(x - 2,300) \\
p - 2.48 &= 0.005x - 11.5 \\
p &= 0.005x - 9.02
\end{align*}
\]

Check:

We check the price–supply equation by verifying that each of the given points satisfies the equation:

\[
\begin{align*}
(2,480, 3.38) & \quad \text{Check:} \\
3.38 &= 0.005(2,480) - 9.02 \\
3.38 &= 2.48 \\
(2,300, 2.48) & \quad \text{Check:} \\
2.48 &= 0.005(2,300) - 9.02 \\
2.48 &= 1.48
\end{align*}
\]

(B) From Table 3, \((2,300, 3.38)\) and \((2,390, 2.48)\) are two points on the linear demand equation.

\[
m = \frac{3.38 - 2.48}{2,300 - 2,390} = \frac{0.9}{-90} = -0.01
\]

\[
\begin{align*}
p - p_1 &= m(x - x_1) \\
p - 2.48 &= -0.01(x - 2,390) \\
p - 2.48 &= -0.01x + 23.9 \\
p &= -0.01x + 26.38
\end{align*}
\]

Check:

\[
\begin{align*}
(2,300, 3.38) & \quad \text{Check:} \\
3.38 & = -0.01(2,300) + 26.38 \\
3.38 & = 2.48 \\
(2,390, 2.48) & \quad \text{Check:} \\
2.48 & = -0.01(2,390) + 26.38 \\
2.48 & = 1.48
\end{align*}
\]

(C) From part (A), we plot the points \((2,480, 3.38)\) and \((2,300, 2.48)\) and then draw a line through them. We do the same with the points \((2,300, 3.38)\) and \((2,390, 2.48)\) from part (B) (Fig. 7). (Note that we restricted the axes to intervals that contain these data points—a common practice in applications.)

\[\text{FIGURE 7} \text{ Graphs of supply and demand equations}\]
To find the intersection point of these lines, we first find \( x \) by solving

\[
\begin{align*}
\text{Price–Supply} & : \quad 0.005x - 9.02 = -0.01x + 26.38 \\
\text{Price–Demand} & : \quad 0.015x = 35.4
\end{align*}
\]

\[ x = 2360 \text{ million bushels} \]

Then we use the price–supply equation to find \( p \) when \( x = 2360 \):

\[
\begin{align*}
p & = 0.005x - 9.02 \\
p & = 0.005(2360) - 9.02 = 2.78
\end{align*}
\]

As a check, we use the price–demand equation to find \( p \) when \( x = 2360 \):

\[
\begin{align*}
p & = -0.01x + 26.38 \\
p & = -0.01(2360) + 26.38 = 2.78
\end{align*}
\]

The lines intersect at \((2360, 2.87)\). See Figure 7.

In Figure 8, we use \texttt{INTERSECT} on a graphing calculator to find the intersection point.

In a free competitive market, the price of a product is determined by the relationship between supply and demand. The price tends to stabilize at the point of intersection of the demand and supply equations. This point is called the \textit{equilibrium point}, the corresponding price is called the \textit{equilibrium price}, and the common value of supply and demand is called the \textit{equilibrium quantity}. All of these are illustrated in Figure 7.

**MATCHED PROBLEM 8**

Use the data in Table 4 to find

(A) A linear supply equation of the form \( p = mx + b \).

(B) A linear demand equation of the form \( p = mx + b \).

(C) The equilibrium point.

<table>
<thead>
<tr>
<th>Year</th>
<th>Supply (mil bu)</th>
<th>Demand (mil bu)</th>
<th>Price ($/bu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>1980</td>
<td>2,420</td>
<td>3.00</td>
</tr>
<tr>
<td>1995</td>
<td>2,180</td>
<td>2,380</td>
<td>4.60</td>
</tr>
</tbody>
</table>

Source: The ProExporter Network®

Answers to Matched Problems

1. \[ y = \frac{1}{3}x - 4 \]

2. \( y \) intercept = \(-4\), \( x \) intercept = \(3\)
Section 1-2  Graphs and Lines  25

3. (A)  
   (B) Horizontal line: y = 2; vertical line: x = -8

4. (A) 0  (B) -4  (C) Not defined  (D) 1

5. \( y = \frac{1}{2}x - 1 \)

6. (A) \( 2x - 3y = 18 \)  
   (B) \( y = 3x - 9 \)

7. (A) \( m = 40 \)  
   (B) \( C = 40x + 250 \)

8. (A) \( p = 0.008x - 12.84 \)  
   (B) \( p = -0.04x + 99.8 \)  
   (C) (2347, 5.93)

Exercise 1-2

A  Problems 1–4 refer to graphs (A)–(D).

1. Identify the graph(s) of lines with a negative slope.
2. Identify the graph(s) of lines with a positive slope.
3. Identify the graph(s) of any lines with slope zero.
4. Identify the graph(s) of any lines with undefined slope.
   What can you say about their slopes?

In Problems 5–8, sketch a graph of each equation in a rectangular coordinate system.
5. \( y = 2x - 3 \)
6. \( y = \frac{x}{2} + 1 \)
7. \( 2x + 3y = 12 \)
8. \( 8x - 37 = 24 \)

In Problems 9–12, find the slope and y intercept of the graph of each equation.
9. \( y = 3x + 1 \)
10. \( y = \frac{x}{5} - 2 \)
11. \( y = -\frac{x}{2} - 6 \)
12. \( y = 0.7x + 5 \)

In Problems 13–16, write an equation of the line with the indicated slope and y intercept.
13. Slope = -2  y intercept = 3
14. Slope = \( \frac{3}{4} \)  y intercept = -5
15. Slope = \( \frac{4}{3} \)  y intercept = -4
16. Slope = -5  y intercept = 9
In Problems 17–20, use the graph of each line to find the x intercept, y intercept, and slope. Write the slope-intercept form of the equation of the line.

17. 

18. 

19. 

20. 

Sketch a graph of each equation or pair of equations in Problems 21–26 in a rectangular coordinate system.

21. 

22. 

23. 

24. 

25. 

26. 

In Problems 27–30, find the slope of the graph of each equation.

27. 

28. 

29. 

30. 

31. Given Ax + By = 12, graph each of the following three cases in the same coordinate system.

(A) A = 2 and B = 0
(B) A = 0 and B = 3
(C) A = 3 and B = 4

32. Given Ax + By = 24, graph each of the following three cases in the same coordinate system.

(A) A = 6 and B = 0
(B) A = 0 and B = 8
(C) A = 2 and B = 3

33. Graph y = 25x + 200, x ≥ 0.

34. Graph y = 40x + 160, x ≥ 0.

35. (A) Graph y = 1.2x − 4.2 in a rectangular coordinate system.
(B) Find the x and y intercepts algebraically to one decimal place.
(C) Graph y = 1.2x − 4.2 in a graphing utility.
(D) Find the x and y intercepts to one decimal place using TRACE and the zero command on your graphing utility.

36. (A) Graph y = −0.8x + 5.2 in a rectangular coordinate system.
(B) Find the x and y intercepts algebraically to one decimal place.
(C) Graph y = −0.8x + 5.2 in a graphing utility.
(D) Find the x and y intercepts to one decimal place using TRACE and the zero command on your graphing utility.
(E) Using the results of parts (A) and (B) or (C) and (D), find the solution set for the linear inequality

−0.8x + 5.2 < 0

In Problems 37–40, write the equations of the vertical and horizontal lines through each point.

37. (4, −3) 
38. (−5, 6) 
39. (−1.5, −3.5) 
40. (2.6, 3.8) 

In Problems 41–46, write the equation of the line through each indicated point with the indicated slope. Write the final answer in the form y = mx + b.

41. m = −4; (2, −3) 
42. m = −6; (−4, 1) 
43. m = 2; (−4, −5) 
44. m = 2; (−6, 2) 
45. m = 0; (−1.5, 4.6) 
46. m = 0; (3.1, −2.7) 

In Problems 47–54,

(A) Find the slope of the line that passes through the given points.

(B) Find the standard form of the equation of the line.

47. (2, 5) and (5, 7) 
48. (1, 2) and (3, 5) 
49. (−2, −1) and (2, −6) 
50. (2, 3) and (−3, 7) 
51. (5, 3) and (5, −3) 
52. (1, 4) and (0, 4) 
53. (−2, 5) and (3, 5) 
54. (2, 0) and (2, −3) 

55. Discuss the relationship among the graphs of the lines with equation y = mx + 2, where m is any real number.

56. Discuss the relationship among the graphs of the lines with equation y = −0.5x + b, where b is any real number.
57. **Simple interest.** If $P$ (the principal) is invested at an interest rate of $r$, then the amount $A$ that is due after $t$ years is given by

$$ A = Prt + P $$

If $100$ is invested at $6\%$ ($r = 0.06$), then

$$ A = 6t + 100, t \geq 0 $$

(A) What will $100$ amount to after 5 years? After 20 years?

(B) Sketch a graph of $A = 6t + 100$ for $0 \leq t \leq 20$.

(C) Find the slope of the graph and interpret verbally.

58. **Simple interest.** Use the simple interest formula from Problem 57. If $1,000$ is invested at $7.5\%$ ($r = 0.075$), then

$$ A = 7.5t + 1,000, t \geq 0 $$

(A) What will $1,000$ amount to after 5 years? After 20 years?

(B) Sketch a graph of $A = 7.5t + 1,000$ for $0 \leq t \leq 20$.

(C) Find the slope of the graph and interpret verbally.

59. **Cost analysis.** A doughnut shop has a fixed cost of $124 per day and a variable cost of $0.12 per doughnut. Find the total daily cost of producing $x$ doughnuts. How many doughnuts can be produced for a total daily cost of $250? 

60. **Cost analysis.** A small company manufactures picnic tables. The weekly fixed cost is $1,200 and the variable cost is $45 per table. Find the total daily cost of producing $x$ picnic tables. How many picnic tables can be produced for a total weekly cost of $4,800? 

61. **Cost analysis.** A plant can manufacture 80 golf clubs per day for a total daily cost of $7,647 and 100 golf clubs per day for a total daily cost of $9,147.

(A) Assuming that daily cost and production are linearly related, find the total daily cost of producing $x$ golf clubs.

(B) Graph the total daily cost for $0 \leq x \leq 200$.

(C) Interpret the slope and $y$-intercept of this cost equation.

62. **Cost analysis.** A plant can manufacture 50 tennis rackets per day for a total daily cost of $3,855 and 60 tennis rackets per day for a total daily cost of $4,245.

(A) Assuming that daily cost and production are linearly related, find the total daily cost of producing $x$ tennis rackets.

(B) Graph the total daily cost for $0 \leq x \leq 100$.

(C) Interpret the slope and $y$-intercept of this cost equation.

63. **Business—Markup policy.** A drug store sells a drug costing $85 for $112 and a drug costing $175 for $238.

(A) If the markup policy of the drug store is assumed to be linear, write an equation that expresses retail price $R$ in terms of cost $C$ (wholesale price).

(B) What does a store pay (to the nearest dollar) for a drug that retails for $185? 

64. **Business—Markup policy.** A clothing store sells a shirt costing $20 for $33 and a jacket costing $60 for $93.

(A) If the markup policy of the store is assumed to be linear, write an equation that expresses retail price $R$ in terms of cost $C$ (wholesale price).

(B) What does a store pay for a suit that retails for $240? 

65. **Business—Depreciation.** A farmer buys a new tractor for $157,000 and assumes that it will have a trade-in value of $82,000 after 10 years. The farmer uses a constant rate of depreciation (commonly called straight-line depreciation—one of several methods permitted by the IRS) to determine the annual value of the tractor.

(A) Find a linear model for the depreciated value $V$ of the tractor $t$ years after it was purchased.

(B) What is the depreciated value of the tractor after 6 years?

(C) When will the depreciated value fall below $70,000? 

(D) Graph $V$ for $0 \leq t \leq 20$ and illustrate the answers from parts (A) and (B) on the graph.

66. **Business—Depreciation.** A charter fishing company buys a new boat for $224,000 and assumes that it will have a trade-in value of $115,200 after 16 years.

(A) Find a linear model for the depreciated value $V$ of the boat $t$ years after it was purchased.

(B) What is the depreciated value of the tractor after 10 years?

(C) When will the depreciated value fall below $100,000? 

(D) Graph $V$ for $0 \leq t \leq 30$ and illustrate the answers from (A) and (B) on the graph.

67. **Boiling point.** The temperature at which water starts to boil is called its boiling point and is linearly related to the altitude. Water boils at 212°F at sea level and at 193.6°F at an altitude of 10,000. (Source: biggreenegg.com)

(A) Find a relationship of the form $T = mx + b$ where $T$ is degrees Fahrenheit and $x$ is altitude in thousands of feet.

(B) Find the boiling point at an altitude of 3,500.

(C) Find the altitude if the boiling point is 200°F.

(D) Graph $T$ and illustrate the answers to (B) and (C) on the graph.

68. **Boiling point.** The temperature at which water starts to boil is also linearly related to barometric pressure. Water boils at 212°F at a pressure of 29.9 inHg (inches of mercury) and at 191°F at a pressure of 28.4 inHg. (Source: biggreenegg.com)

(A) Find a relationship of the form $T = mx + b$, where $T$ is degrees Fahrenheit and $x$ is pressure in inches of mercury.

(B) Find the boiling point at a pressure of 31 inHg.

(C) Find the pressure if the boiling point is 199°F.

(D) Graph $T$ and illustrate the answers to (B) and (C) on the graph.

69. **Flight conditions.** In stable air, the air temperature drops about 3.6°F for each 1,000-foot rise in altitude. (Source: Federal Aviation Administration)

(A) If the temperature at sea level is 70°F, write a linear equation that expresses temperature $T$ in terms of altitude $A$ in thousands of feet.

(B) At what altitude is the temperature 34°F?
28 CHAPTER 1 Linear Equations and Graphs

airspeed by observing the indicated airspeed and adding to it about 1.6% for every 1,000 feet of altitude. (Source: Megginsion Technologies Ltd.)

(A) If a pilot maintains a constant reading of 200 miles per hour on the airspeed indicator as the aircraft climbs from sea level to an altitude of 10,000 feet, write a linear equation that expresses true airspeed $T$ (in miles per hour) in terms of altitude $A$ (in thousands of feet).

(B) What would be the true airspeed of the aircraft at 6,500 feet?

71. Demographics. The average number of persons per household in the United States has been shrinking steadily for as long as statistics have been kept and is approximately linear with respect to time. In 1980, there were about 2.8 persons per household and in 2005, about 2.6. (Source: U.S. Census Bureau)

(A) If $N$ represents the average number of persons per household and $t$ represents the number of years since 1980, write a linear equation that expresses $N$ in terms of $t$.

(B) Use this equation to estimate household size in the year 2030.

72. Demographics. The median household income divides the households into two groups, the half whose income is less than or equal to the median and the half whose income is greater than the median. The median household income in the United States grew from about $34,000 in 1980 to about $45,000 in 2005. (Source: U.S. Census Bureau)

(A) If $I$ represents the median household income and $t$ represents the number of years since 1980, write a linear equation that expresses $I$ in terms of $t$.

(B) Use this equation to estimate median household income in the year 2030.

73. Cigarette smoking. The percentage of female cigarette smokers in the United States declined from 21% in 2000 to 18.5% in 2004. (Source: Centers for Disease Control)

(A) Find a linear equation relating percentage of female smokers ($f$) to years since 2000 ($t$).

(B) Find the year in which the percentage of female smokers falls below 15%.

(C) Graph the equation for $0 \leq t \leq 15$ and illustrate the answer to part (B) on the graph.

74. Cigarette smoking. The percentage of male cigarette smokers in the United States declined from 25.7% in 2000 to 23.4% in 2004. (Source: Centers for Disease Control)

(A) Find a linear equation relating percentage of male smokers ($m$) to years since 2000 ($t$).

(B) Find the year in which the percentage of male smokers falls below 20%.

(C) Graph the equation for $0 \leq t \leq 15$ and illustrate the answer to part (B) on the graph.

75. Supply and demand. Use the barley market data in Table 5 to find

(A) A linear supply equation of the form $p = mx + b$.

(B) A linear demand equation of the form $p = mx + b$.

(C) The equilibrium point.

(D) Graph the supply equation, demand equation, and equilibrium point in the same coordinate system.

Table 5: U.S. Barley Supply and Demand

<table>
<thead>
<tr>
<th>Year</th>
<th>Supply (mil bu)</th>
<th>Demand (mil bu)</th>
<th>Price ($/bu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>7,500</td>
<td>7,900</td>
<td>2.28</td>
</tr>
<tr>
<td>1991</td>
<td>7,900</td>
<td>8,800</td>
<td>2.37</td>
</tr>
</tbody>
</table>

Source: U.S. Department of Agriculture

76. Supply and demand. Use the corn market data in Table 6 to find

(A) A linear supply equation of the form $p = mx + b$.

(B) A linear demand equation of the form $p = mx + b$.

(C) The equilibrium point.

(D) Graph the supply equation, demand equation, and equilibrium point in the same coordinate system.

Table 6: U.S. Corn Supply and Demand

<table>
<thead>
<tr>
<th>Year</th>
<th>Supply (mil bu)</th>
<th>Demand (mil bu)</th>
<th>Price ($/bu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>9,800</td>
<td>9,300</td>
<td>1.94</td>
</tr>
<tr>
<td>1999</td>
<td>9,400</td>
<td>9,500</td>
<td>1.82</td>
</tr>
</tbody>
</table>

Source: The ProExporter Network®

77. Physics. Hooke’s law states that the relationship between the stretch $s$ of a spring and the weight $w$ causing the stretch is linear (a principle upon which all spring scales are constructed). For a particular spring, a 5-pound weight causes a stretch of 2 inches, while with no weight the stretch of the spring is 0.

(A) Find a linear equation that expresses $s$ in terms of $w$.

(B) What is the stretch for a weight of 20 pounds?

(C) What weight will cause a stretch of 3.6 inches?

78. Physics. The distance $d$ between a fixed spring and the floor is a linear function of the weight $w$ attached to the bottom of the spring. The bottom of the spring is 18 inches from the floor when the weight is 3 pounds and 10 inches from the floor when the weight is 5 pounds.

(A) Find a linear equation that expresses $d$ in terms of $w$.

(B) Find the distance from the bottom of the spring to the floor if no weight is attached.

(C) Find the smallest weight that will make the bottom of the spring touch the floor. (Ignore the height of the weight.)

79. Energy consumption. Table 7 lists U.S. oil imports as a percentage of total energy consumption for selected years.

Table 7: U.S. Oil Imports

<table>
<thead>
<tr>
<th>Year</th>
<th>Oil Imports (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>9</td>
</tr>
<tr>
<td>1970</td>
<td>12</td>
</tr>
<tr>
<td>1980</td>
<td>21</td>
</tr>
<tr>
<td>1990</td>
<td>24</td>
</tr>
<tr>
<td>2000</td>
<td>29</td>
</tr>
</tbody>
</table>

Source: Energy Information Administration

*We use percentage because energy is measured in quadrillion BTUs (British Thermal Units) and one quadrillion is 1,000,000,000,000,000. Percentages are easier to comprehend.
Let $x$ represent years since 1960 and $y$ represent the corresponding percentage of oil imports.

(A) Find the equation of the line through $(0, 9)$ and $(40, 29)$, the first and last data points in Table 7.

(B) Find the equation of the line through $(0, 9)$ and $(10, 12)$, the first and second data points in Table 7.

(C) Graph the lines from parts (A) and (B) and the data points $(x, y)$ from Table 7 in the same coordinate system.

(D) Use each equation to predict oil imports as a percentage of total consumption in 2020.

(E) Which of the two lines seems to better represent the data in Table 7? Discuss.

---

80. **Energy production.** Table 8 lists U.S. crude oil production as a percentage of total U.S. energy production for selected years. Let $x$ represent years since 1960 and $y$ represent the corresponding percentage of oil production.

(A) Find the equation of the line through $(0, 35)$ and $(40, 17)$, the first and last data points in Table 8.

(B) Find the equation of the line through $(0, 35)$ and $(10, 32)$, the first and second data points in Table 8.

(C) Graph the lines from parts (A) and (B) and the data points $(x, y)$ from Table 8 in the same coordinate system.

(D) Use each equation to predict oil imports as a percentage of total consumption in 2020.

(E) Which of the two lines seems to better represent the data in Table 8? Discuss.

---

**Section 1-3 LINEAR REGRESSION**

- **Slope as a Rate of Change**
- **Linear Regression**

Mathematical modeling is the process of using mathematics to solve real-world problems. This process can be broken down into three steps (Fig. 1):

**Step 1.** Construct the mathematical model, a problem whose solution will provide information about the real-world problem.

**Step 2.** Solve the mathematical model.

**Step 3.** Interpret the solution to the mathematical model in terms of the original real-world problem.

In more complex problems, this cycle may have to be repeated several times to obtain the required information about the real-world problem. In this section we will discuss one of the simplest mathematical models, a linear equation. With the aid of a graphing calculator or computer, we also will learn how to analyze a linear model based on real-world data.

- **Slope as a Rate of Change**

If $x$ and $y$ are related by the equation $y = mx + b$, where $m$ and $b$ are constants with $m \neq 0$, then $x$ and $y$ are *linearly related*. If $(x_1, y_1)$ and $(x_2, y_2)$ are two distinct points on this line, then the slope of the line is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
In applications, ratio (1) is called the \textit{rate of change} of $y$ with respect to $x$. Since the slope of a line is unique, the rate of change of two linearly related variables is constant. Here are some examples of familiar rates of change: miles per hour, revolutions per minute, price per pound, passengers per plane, and so on. If the relationship between $x$ and $y$ is not linear, ratio (1) is called the \textit{average rate of change} of $y$ with respect to $x$.

\begin{equation}
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x}
\end{equation}

Estimating Body Surface Area

Appropriate doses of medicine for both animals and humans are often based on body surface area (BSA). Since weight is much easier to determine than BSA, veterinarians use the weight of an animal to estimate BSA. The following linear equation expresses BSA for canines in terms of weight:

$$a = 16.12w + 375.6$$

where $a$ is BSA in square inches and $w$ is weight in pounds.

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a one pound increase in weight?

\textbf{SOLUTION}

(A) The rate of change BSA with respect to weight is 16.12 square inches per pound.

(B) Since slope is the ratio of rise to run, increasing $w$ by 1 pound (run) increases $a$ by 16.12 square inches (rise).

\textbf{MATCHED PROBLEM 1}

The equation $a = 28.55w + 118.7$ expresses BSA for felines in terms of weight, where $a$ is BSA in square inches and $w$ is weight in pounds.

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a one pound increase in weight?

Explore & Discuss

As illustrated in Example 1A, the slope $m$ of a line with equation $y = mx + b$ has two interpretations:

1. $m$ is the rate of change of $y$ with respect to $x$.
2. Increasing $x$ by one unit will change $y$ by $m$ units.

How are these two interpretations related?

Parachutes are used to deliver cargo to areas that cannot be reached by other means of conveyance. The \textit{rate of descent} of the cargo is the rate of change of altitude with respect to time. The absolute value of the rate of descent is called the \textit{speed} of the cargo. At low altitudes, the altitude of the cargo and the time in the air are linearly related. The appropriate rate of descent varies widely with the item. Bulk food (rice, flour, beans, etc.) and clothing can tolerate nearly any rate of descent under 40 ft/sec. Machinery and electronics (pumps, generators, radios, etc.) should generally be dropped at 15 ft/sec or less. Butler Tactical Parachute Systems, Roanoke, Virginia, manufactures a variety of canopies for dropping cargo. The following example uses information taken from one of the company’s brochures.

\textbf{EXAMPLE 2}

\textbf{Finding the Rate of Descent}

A 100-pound cargo of delicate electronic equipment is dropped from an altitude of 2,880 feet and lands 200 seconds later.

(A) Find a linear model relating altitude $a$ (in feet) and time in the air $t$ (in seconds).

(B) How fast is the cargo moving when it lands?

\* Based on data from Veterinary Oncology Consultants, PTY LTD.
Section 1-3  Linear Regression

(A) If \( a = mt + b \) is the linear equation relating altitude \( a \) and time in air \( t \), then the graph of this equation must pass through the following points:

\[
(t_1, a_1) = (0, 2,880) \quad \text{Cargo is dropped from plane.} \\
(t_2, a_2) = (200, 0) \quad \text{Cargo lands.}
\]

The slope of this line is

\[
m = \frac{a_2 - a_1}{t_2 - t_1} = \frac{0 - 2,880}{200 - 0} = -14.4
\]

and the equation of this line is

\[
a - 0 = -14.4(t - 200) \\
a = -14.4t + 2,880
\]

(B) The rate of descent is the slope \( m = -14.4 \), so the speed of the cargo at landing is \( | -14.4 | = 14.4 \) ft/sec.

MATCHED PROBLEM 2

A 400-pound load of grain is dropped from an altitude of 2,880 feet and lands 80 seconds later.

(A) Find a linear model relating altitude \( a \) (in feet) and time in the air \( t \) (in seconds).

(B) How fast is the cargo moving when it lands?

Linear Regression

In real-world applications we often encounter numerical data in the form of a table. The very powerful mathematical tool regression analysis, can be used to analyze numerical data. In general, regression analysis is a process for finding a function that provides a useful model for a set of data points. Graphs of equations are often called curves and regression analysis is also referred to as curve fitting. In the next example, we use a linear model obtained by using linear regression on a graphing calculator.

EXAMPLE 3  Diamond Prices

Prices for round-shaped diamonds taken from an online trader are given in Table 1.

<table>
<thead>
<tr>
<th>Weight (carats)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$2,790</td>
</tr>
<tr>
<td>0.6</td>
<td>$3,191</td>
</tr>
<tr>
<td>0.7</td>
<td>$3,694</td>
</tr>
<tr>
<td>0.8</td>
<td>$4,154</td>
</tr>
<tr>
<td>0.9</td>
<td>$5,018</td>
</tr>
<tr>
<td>1.0</td>
<td>$5,898</td>
</tr>
</tbody>
</table>

Source: www.tradeshop.com

(A) A linear model for the data in Table 1 is given by

\[
p = 6,140c - 480
\]

where \( p \) is the price of a diamond weighing \( c \) carats. (We will discuss the source of models like this later in this section.) Plot the points in Table 1 on a Cartesian coordinate system, producing a scatter plot, and graph the model on the same axes.
(B) Interpret the slope of the model in (2).

(C) Use the model to estimate the cost of a 0.85-carat diamond and the cost of a 1.2-carat diamond. Round answers to the nearest dollar.

(D) Use the model to estimate the weight of a diamond (to two decimal places) that sells for $4,000.

**SOLUTION**

(A) A scatter plot is simply a graph of the points in Table 1 (Fig. 2A). To add the graph of the model to the scatter plot, we find any two points that satisfy equation (2) [we choose (0.4, 1,976) and (1.1, 6,274)]. Plotting these points and drawing a line through them gives us Figure 2B.

![Scatter plot](image)

![Linear model](image)

**FIGURE 2**

(B) The rate of change of the price of a diamond with respect to its weight is 6,140. Increasing the weight by one carat will increase the price by about $6,140.

(C) The graph of the model (Fig. 2B) does not pass through any of the points in the scatter plot, but it comes close to all of them. [Verify this by evaluating equation (2) at \( c = 0.5, 0.6, \ldots, 1 \).] So we can use equation (2) to approximate points not in Table 1.

\[
\begin{align*}
  c & = 0.85 & c & = 1.2 \\
  p & \approx 6,140(0.85) - 480 & p & \approx 6,140(1.2) - 480 \\
  p & = 4,739 & p & = 6,888
\end{align*}
\]

A 0.85-carat diamond will cost about $4,739 and a 1.2-carat diamond will cost about $6,888.
(D) To find the weight of a $4,000 diamond, we solve the following equation for $c$:

$$6,140c - 480 = 4,000$$

Add 480 to both sides.

$$6,140c = 4,480$$

Divide both sides by 6,140.

$$c = \frac{4,480}{6,140} \approx 0.73$$

Rounded to two decimal places.

A $4,000 diamond will weigh about 0.73 carats.

MATCHED PROBLEM 3

Prices for emerald-shaped diamonds taken from an online trader are given in Table 2. Repeat Example 3 for this data with the linear model

$$p = 5,600c - 1,100$$

where $p$ is the price of an emerald-shaped diamond weighing $c$ carats.

<table>
<thead>
<tr>
<th>Weight (carats)</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$1,677</td>
</tr>
<tr>
<td>0.6</td>
<td>$2,353</td>
</tr>
<tr>
<td>0.7</td>
<td>$2,718</td>
</tr>
<tr>
<td>0.8</td>
<td>$3,218</td>
</tr>
<tr>
<td>0.9</td>
<td>$3,982</td>
</tr>
<tr>
<td>1.0</td>
<td>$4,510</td>
</tr>
</tbody>
</table>

Source: www.tradeshop.com

The model we used in Example 3 was obtained by using a technique called linear regression and the model is called the regression line. This technique produces a line that is the best fit for a given data set. We will not discuss the theory behind this technique, nor the meaning of best fit. Although you can find a linear regression line by hand, we prefer to leave the calculations to a graphing calculator or a computer. Don’t be concerned if you don’t have either of these electronic devices. We will supply the regression model in most of the applications we discuss, as we did in Example 3.

Explore & Discuss 2

We used linear regression to produce the model in Example 3. If you have a graphing calculator that supports linear regression, then you can find this model. The linear regression process varies greatly from one calculator to another. Consult the user’s manual* for the details of linear regression. The screens in Figure 3 are related to the construction of the model in Example 3 on a Texas Instruments TI-84 Plus.

(A) Produce similar screens on your graphing calculator.

(B) Do the same for Matched Problem 3.

* Manuals for most graphing calculators are readily available on the Internet.
In Example 3, we used the regression model to approximate points that were not given in Table 1 but would fit between points in the table. This process is called **interpolation**. In the next example, we use a regression model to approximate points outside the given data set. This process is called **extrapolation**, and the approximations are often referred to as **predictions**.

### Example 4

**Carbon Monoxide Emissions** Table 3 contains information about carbon monoxide emissions in recent years. The transportation emissions include emissions from all trains, planes, automobiles, and any other types of transportation. The linear regression model for the transportation emissions (after rounding) is

\[ C = 152 - 4t \]

where \( C \) is the carbon monoxide emissions (in millions of short tons*) and \( t \) is time in years with \( t = 0 \) corresponding to 1980.

(A) Interpret the slope of the regression line as a rate of change.

(B) Use the regression model to predict the emissions from the transportation sector in 2010.

#### Solution

(A) The slope \( m = -4 \) is the rate of change of emissions with respect to time. Since the slope is negative and the emissions are given in millions of tons, the emissions are decreasing at a rate of 4,000,000 tons per year.

(B) If \( t = 30 \), then

\[ C = 152 - 4(30) = 32 \quad \text{or} \quad 32,000,000 \text{ tons} \]

So approximately 32,000,000 tons of carbon monoxide will be emitted in 2010.

### Matched Problem 4

Repeat Example 4 using the linear regression model

\[ C = 0.16t + 38 \]

where \( C \) is the carbon monoxide emissions that are not caused by transportation (in millions of short tons) and time in years with \( t = 0 \) corresponding to 1980.

### Explore & Discuss

Use the Internet or a library to expand the data in Table 3 to include years after 2004. Compare the actual values for the years you find to the model’s predicted values. Discuss the accuracy of predictions given by the regression model as time goes by.

Forest managers must estimate things like growth, volume, yield, and forest potential. One common measure is the diameter of a tree at breast height (Dbh), which is defined as the diameter of the tree at a point 4.5 feet above the ground on the uphill side of the tree. Example 5 is concerned with using Dbh to estimate the height of balsam fir trees.

---

* A short ton (or a U.S. ton) is 2,000 pounds, a long ton (or a U.K. ton) is 2,240 pounds, and a metric tonne is 1,000 kilograms or 2,204.6 pounds.
EXAMPLE 5

Forestry A linear regression model for the height of balsam fir trees is
\[ h = 3.8d + 18.73 \]
where \( d \) is Dbh in inches and \( h \) is the height in feet.

(A) Interpret the slope of this model.

(B) What is the effect of a 1-inch increase in Dbh?

(C) Estimate the height of a balsam fir with a Dbh of 8 inches. Round your answer to the nearest foot.

(D) Estimate the Dbh of a balsam fir that is 30 feet tall. Round your answer to the nearest inch.

SOLUTION

(A) The rate of change of height with respect to breast height diameter is 3.8 feet per inch.

(B) Height increases by 3.8 feet.

(C) We must find \( h \) when \( d = 8 \):
\[
\begin{align*}
  h &= 3.8d + 18.73 \\
  h &= 3.8(8) + 18.73 \\
  h &= 49.13 \approx 49 \text{ ft}
\end{align*}
\]

(D) We must find \( d \) when \( h = 30 \):
\[
\begin{align*}
  h &= 3.8d + 18.73 \\
  30 &= 3.8d + 18.73 \\
  11.27 &= 3.8d \\
  d &= \frac{11.27}{3.8} \approx 3 \text{ in}
\end{align*}
\]

The data used to produce the regression model in Example 5 are from a stand of trees in the Jack Haggerty Forest at Lakeland University in Canada (Table 4). We used the popular spreadsheet Excel to produce a scatter plot of the data in Table 4 and to find the regression model (Fig. 4).
### TABLE 4  Balsam Fir

<table>
<thead>
<tr>
<th>Dbh (in)</th>
<th>Height (ft)</th>
<th>Dbh (in)</th>
<th>Height (ft)</th>
<th>Dbh (in)</th>
<th>Height (ft)</th>
<th>Dbh (in)</th>
<th>Height (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>51.8</td>
<td>6.4</td>
<td>44.0</td>
<td>3.1</td>
<td>19.7</td>
<td>4.6</td>
<td>26.6</td>
</tr>
<tr>
<td>8.6</td>
<td>50.9</td>
<td>4.4</td>
<td>46.9</td>
<td>7.1</td>
<td>55.8</td>
<td>4.8</td>
<td>33.1</td>
</tr>
<tr>
<td>5.7</td>
<td>49.2</td>
<td>6.5</td>
<td>52.2</td>
<td>6.3</td>
<td>32.8</td>
<td>3.1</td>
<td>28.5</td>
</tr>
<tr>
<td>4.9</td>
<td>46.3</td>
<td>4.1</td>
<td>46.9</td>
<td>2.4</td>
<td>26.2</td>
<td>3.2</td>
<td>29.2</td>
</tr>
<tr>
<td>6.4</td>
<td>44.3</td>
<td>8.8</td>
<td>51.2</td>
<td>2.5</td>
<td>29.5</td>
<td>5.0</td>
<td>34.1</td>
</tr>
<tr>
<td>4.1</td>
<td>46.9</td>
<td>5.0</td>
<td>36.7</td>
<td>6.9</td>
<td>45.9</td>
<td>3.0</td>
<td>28.2</td>
</tr>
<tr>
<td>1.7</td>
<td>13.1</td>
<td>4.9</td>
<td>34.1</td>
<td>2.4</td>
<td>32.8</td>
<td>4.8</td>
<td>33.8</td>
</tr>
<tr>
<td>1.8</td>
<td>19.0</td>
<td>3.8</td>
<td>32.2</td>
<td>4.3</td>
<td>39.4</td>
<td>4.4</td>
<td>35.4</td>
</tr>
<tr>
<td>3.2</td>
<td>20.0</td>
<td>5.5</td>
<td>49.2</td>
<td>7.3</td>
<td>36.7</td>
<td>11.3</td>
<td>55.4</td>
</tr>
<tr>
<td>5.1</td>
<td>46.6</td>
<td>6.3</td>
<td>39.4</td>
<td>10.9</td>
<td>51.5</td>
<td>3.7</td>
<td>32.2</td>
</tr>
</tbody>
</table>

### MATCHED PROBLEM 5

Figure 5 shows the scatter plot for a stand of white spruce trees in the Jack Haggerty Forest at Lakeland University in Canada. A regression model produced by a spreadsheet (Fig. 5), after rounding, is

\[ h = 1.8d + 34 \]

where \( d \) is Dbh in inches and \( h \) is the height in feet.

(A) Interpret the slope of this model.

(B) What is the effect of a 1-inch increase in Dbh?

(C) Estimate the height of a white spruce with a Dbh of 10 inches. Round your answer to the nearest foot.

(D) Estimate the Dbh of a white spruce that is 65 feet tall. Round your answer to the nearest inch.

### Answers to Matched Problems

1. (A) The rate of change BSA with respect to weight is 28.55 square inches per pound.

   (B) Increasing \( w \) by 1 pound increases \( a \) by 28.55 square inches.

2. (A) \( a = -36t + 2,880 \)  

   (B) 36 ft/sec
Exercise 1-3

Applications

1. Ideal weight. In 1983 Dr. J. D. Robinson published the following estimate of the ideal body weight of a woman:

\[ 49 \text{ kg} + 1.7 \text{ kg for each inch over 5 feet} \]

(A) Find a linear model for Robinson’s estimate of the ideal weight of a woman using \( w \) for ideal body weight (in kilograms) and \( h \) for height over 5 feet (in inches).

(B) Interpret the slope of the model.

(C) If a woman is 5’4” tall, what does the model predict her weight to be?

(D) If a woman weighs 60 kilograms, what does the model predict her height to be?

2. Ideal weight. Dr. J. D. Robinson also published the following estimate of the ideal body weight of a man:

\[ 52 \text{ kg} + 1.9 \text{ kg for each inch over 5 feet} \]

(A) Find a linear model for Robinson’s estimate of the ideal weight of a man using \( w \) for ideal body weight (in kilograms) and \( h \) for height over 5 feet (in inches).

(B) Interpret the slope of the model.

(C) If a man is 5’8” tall, what does the model predict his weight to be?

(D) If a man weighs 70 kilograms, what does the model predict his height to be?

3. Underwater pressure. At sea level, the weight of the atmosphere exerts a pressure of 14.7 pounds per square inch, commonly referred to as 1 atmosphere of pressure. As an object descends in water, pressure \( P \) and depth \( d \) are linearly related. In salt water, the pressure at a depth of 33 feet is 2 atmospheres or 29.4 pounds per square inch.

(A) Find a linear model that relates pressure \( P \) (in pounds per square inch) to depth \( d \) (in feet).

(B) Interpret the slope of the model.

(C) Find the pressure at a depth of 50 feet.

(D) Find the depth at which the pressure is 4 atmospheres.

4. Underwater pressure. Refer to Problem 3. In fresh water, the pressure at a depth of 34 feet is 2 atmospheres or 29.4 pounds per square inch.

(A) Find a linear model that relates pressure \( P \) (in pounds per square inch) to depth \( d \) (in feet).

(B) Interpret the slope of the model.

(C) Find the pressure at a depth of 50 feet.

(D) Find the depth at which the pressure is 4 atmospheres.

5. Rate of descent—Parachutes. At low altitudes the altitude of a parachutist and time in the air are linearly related. A jump at 2,880 ft using the U.S. Army’s T-10 parachute system lasts 120 seconds.

(A) Find a linear model relating altitude \( a \) (in feet) and time in the air \( t \) (in seconds).
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(B) Find the rate of descent for a T-10 system.
(C) Find the speed of the parachutist at landing.

6. Rate of descent—Parachutes. The U.S. Army is considering a new parachute, the Advanced Tactical Parachute System (ATPS). A jump at 2,880 ft using the ATPS system lasts 180 seconds.
(A) Find a linear model relating altitude \(a\) (in feet) and time in the air \(t\) (in seconds).
(B) Find the rate of descent for an ATPS system parachute.
(C) Find the speed of the parachutist at landing.

7. Speed of sound. The speed of sound through air is linearly related to the temperature of the air. If sound travels at 331 m/sec at 0°C and at 343 m/sec at 20°C, construct a linear model relating the speed of sound \(s\) and the air temperature \(t\). Interpret the slope of this model.

\( s = at + b \)

where \(a\) and \(b\) are constants.

\( a = \frac{343 - 331}{20 - 0} = \frac{12}{20} = 0.6 \)

\( b = 331 - 0.6 \times 0 = 331 \)

\( s = 0.6t + 331 \)

8. Speed of sound. The speed of sound through sea water is linearly related to the temperature of the water. If sound travels at 1,403 m/sec at 0°C and at 1,481 m/sec at 20°C, construct a linear model relating the speed of sound \(s\) and the air temperature \(t\). Interpret the slope of this model.

\( s = at + b \)

where \(a\) and \(b\) are constants.

\( a = \frac{1481 - 1403}{20 - 0} = \frac{78}{20} = 3.9 \)

\( b = 1403 - 3.9 \times 0 = 1403 \)

\( s = 3.9t + 1403 \)

9. Energy production. Table 5 lists U.S. fossil fuel production as a percentage of total energy production for selected years. A linear regression model for this data is

\( y = -0.36x + 94.6 \)

where \(x\) represents years since 1960 and \(y\) represents the corresponding percentage of oil imports.

<table>
<thead>
<tr>
<th>TABLE 5 U.S. Fossil Fuel Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1960</td>
</tr>
<tr>
<td>1970</td>
</tr>
<tr>
<td>1980</td>
</tr>
<tr>
<td>1990</td>
</tr>
<tr>
<td>2000</td>
</tr>
</tbody>
</table>

Source: Energy Information Administration

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) Interpret the slope of the model.
(C) Use the model to predict fossil fuel production in 2010.
(D) Use the model to estimate the year in which fossil fuel consumption will fall below 80% of total energy consumption.

10. Energy consumption. Table 6 lists U.S. fossil fuel consumption as a percentage of total energy consumption for selected years. A linear regression model for this data is

\( y = -0.22x + 94 \)

where \(x\) represents years since 1960 and \(y\) represents the corresponding percentage of fossil fuel consumption.

<table>
<thead>
<tr>
<th>TABLE 6 U.S. Fossil Fuel Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>1960</td>
</tr>
<tr>
<td>1970</td>
</tr>
<tr>
<td>1980</td>
</tr>
<tr>
<td>1990</td>
</tr>
<tr>
<td>2000</td>
</tr>
</tbody>
</table>

Source: Energy Information Administration

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) Interpret the slope of the model.
(C) Use the model to predict fossil fuel consumption in 2010.
(D) Use the model to estimate the year in which fossil fuel consumption will fall below 80% of total energy consumption.

11. Cigarette smoking. The data in Table 7 shows that the percentage of female cigarette smokers in the United States declined from 21% in 2000 to 18.5% in 2004.
(A) Applying linear regression to the data for females in Table 7 produces the model

\( f = -0.65t + 21.18 \)

where \(f\) is percentage of female smokers and \(t\) is time in years. Draw a scatter plot of the female smoker data and a graph of the regression model on the same axes for \(0 \leq t \leq 15\).
(B) Estimate the year in which the percentage of female smokers falls below 15% and illustrate the answer on your graph from part (A).

<table>
<thead>
<tr>
<th>TABLE 7 Percentage of Smoking Prevalence among U.S. Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>2000</td>
</tr>
<tr>
<td>2001</td>
</tr>
<tr>
<td>2002</td>
</tr>
<tr>
<td>2003</td>
</tr>
<tr>
<td>2004</td>
</tr>
</tbody>
</table>

Source: Centers for Disease Control

12. Cigarette smoking. The data in Table 7 shows that the percentage of male cigarette smokers in the United States declined from 25.7% in 2000 to 23.4% in 2004.
(A) Applying linear regression to the data for males in Table 7 produces the model

\( m = -0.57t + 25.86 \)

where \(m\) is percentage of male smokers and \(t\) is time in years. Draw a scatter plot of the male smoker data and a graph of the regression model for \(0 \leq t \leq 15\).
(B) Estimate the year in which the percentage of male smokers falls below 20% and illustrate the answer on your graph from part (A).

13. Real estate. Table 8 contains recent average and median purchase prices for a house in Texas. A linear regression model for the average purchase price is

\( y = 5.3x + 145 \)

where \(x\) is years since 2000 and \(y\) is average purchase price (in thousands of dollars).
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14. **Real estate.** A linear regression model for the median purchase price in Table 8 is

\[ y = 4.4x + 114 \]

where \( x \) is years since 2000 and \( y \) is median purchase price (in thousands of dollars).

(A) Plot the median price data and the model on the same axes.
(B) Predict the median price in 2010.
(C) Interpret the slope of the model.

15. **Licensed drivers.** Table 9 contains the state population and the number of licensed drivers in the state (both in millions) for the states with population under 1 million in 2004. The regression model for this data is

\[ y = 0.63x + 0.08 \]

where \( x \) is the state population and \( y \) is the number of licensed drivers in the state.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) If the population of Minnesota in 2004 was about 5 million, use the model to estimate the number of licensed drivers in Minnesota in 2004 to the nearest thousand.
(C) If the number of licensed drivers in Wisconsin in 2004 was about 3.9 million, use the model to estimate the population of Wisconsin in 2004 to the nearest thousand.

16. **Licensed drivers.** Table 10 contains the state population and the number of licensed drivers in the state (both in millions) for the states with population over 10 million in 2004. The regression model for this data is

\[ y = 0.62x + 0.78 \]

where \( x \) is the state population and \( y \) is the number of licensed drivers in the state.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) If the population of Idaho in 2004 was about 1.4 million, use the model to estimate the number of licensed drivers in Idaho in 2004 to the nearest thousand.
(C) If the number of licensed drivers in Rhode Island in 2004 was about 0.74 million, use the model to estimate the population of Rhode Island in 2004 to the nearest thousand.

17. **Revenue.** A linear regression model for the revenue data in Table 11 is

\[ R = 27.6t + 205 \]

where \( R \) is total annual revenue and \( t \) is time since 1/31/02 in years.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) If the population of Minnesota in 2004 was about 5 million, use the model to estimate the number of licensed drivers in Minnesota in 2004 to the nearest thousand.
(C) If the number of licensed drivers in Wisconsin in 2004 was about 3.9 million, use the model to estimate the population of Wisconsin in 2004 to the nearest thousand.

18. **Profit.** A linear regression model for the gross profit data in Table 11 is

\[ R = 6.8t + 44.6 \]

where \( R \) is gross profit and \( t \) is time since 1/31/02 in years.

---

**TABLE 8** Texas Real Estate Prices

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Price (in thousands)</th>
<th>Median Price (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>$146</td>
<td>$112</td>
</tr>
<tr>
<td>2001</td>
<td>$150</td>
<td>$120</td>
</tr>
<tr>
<td>2002</td>
<td>$156</td>
<td>$125</td>
</tr>
<tr>
<td>2003</td>
<td>$160</td>
<td>$128</td>
</tr>
<tr>
<td>2004</td>
<td>$164</td>
<td>$130</td>
</tr>
<tr>
<td>2005</td>
<td>$174</td>
<td>$136</td>
</tr>
</tbody>
</table>

*Source: Real Estate Center at Texas A&M University*

**TABLE 9** Licensed Drivers in 2004

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
<th>Licensed Drivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska</td>
<td>0.66</td>
<td>0.48</td>
</tr>
<tr>
<td>Delaware</td>
<td>0.83</td>
<td>0.53</td>
</tr>
<tr>
<td>Montana</td>
<td>0.93</td>
<td>0.71</td>
</tr>
<tr>
<td>North Dakota</td>
<td>0.63</td>
<td>0.46</td>
</tr>
<tr>
<td>South Dakota</td>
<td>0.77</td>
<td>0.56</td>
</tr>
<tr>
<td>Vermont</td>
<td>0.62</td>
<td>0.55</td>
</tr>
<tr>
<td>Wyoming</td>
<td>0.51</td>
<td>0.38</td>
</tr>
</tbody>
</table>

*Source: Bureau of Transportation Statistics*

**TABLE 10** Licensed Drivers in 2004

<table>
<thead>
<tr>
<th>State</th>
<th>Population</th>
<th>Licensed Drivers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alaska</td>
<td>36</td>
<td>23</td>
</tr>
<tr>
<td>Florida</td>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>Illinois</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>Michigan</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>New York</td>
<td>19</td>
<td>11</td>
</tr>
<tr>
<td>Ohio</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>Pennsylvania</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Texas</td>
<td>22</td>
<td>15</td>
</tr>
</tbody>
</table>

*Source: Bureau of Transportation Statistics*

**TABLE 11** Wal-Mart Stores, Inc.

<table>
<thead>
<tr>
<th>Billions of U.S. Dollars</th>
<th>12 Months Ending 1/31/02</th>
<th>12 Months Ending 1/31/03</th>
<th>12 Months Ending 1/31/04</th>
<th>12 Months Ending 1/31/05</th>
<th>12 Months Ending 1/31/06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Revenue</td>
<td>206</td>
<td>232</td>
<td>259</td>
<td>288</td>
<td>316</td>
</tr>
<tr>
<td>Gross Profit</td>
<td>45</td>
<td>51</td>
<td>58</td>
<td>65</td>
<td>72</td>
</tr>
</tbody>
</table>

*Source: Reuters*
(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) Predict Wal-Mart’s annual gross profit for the period ending 1/31/10.

19. **Freezing temperature.** Ethylene glycol and propylene glycol are liquids used in antifreeze and deicing solutions. Ethylene glycol is listed as a hazardous chemical by the Environmental Protection Agency, while propylene glycol is generally regarded as safe. Table 12 lists the freezing temperature for various concentrations (as a percentage of total weight) of each chemical in a solution used to deice airplanes. A linear regression model for the ethylene glycol data in Table 12 is

\[ E = -0.55T + 31 \]

where \( E \) is the percentage of ethylene glycol in the deicing solution and \( T \) is the temperature at which the solution freezes.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) Use the model to estimate the freezing temperature to the nearest degree of a solution that is 30% ethylene glycol.
(C) Use the model to estimate the percentage of ethylene glycol in a solution that freezes at 15°F.

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>Ethylene Glycol (%Wt.)</th>
<th>Propylene Glycol (%Wt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50</td>
<td>56</td>
<td>58</td>
</tr>
<tr>
<td>-40</td>
<td>53</td>
<td>55</td>
</tr>
<tr>
<td>-30</td>
<td>49</td>
<td>52</td>
</tr>
<tr>
<td>-20</td>
<td>45</td>
<td>48</td>
</tr>
<tr>
<td>-10</td>
<td>40</td>
<td>43</td>
</tr>
<tr>
<td>0</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>29</td>
</tr>
<tr>
<td>20</td>
<td>16</td>
<td>19</td>
</tr>
</tbody>
</table>

Source: T. Labuza, University of Minesota

20. **Freezing temperature.** A linear regression model for the propylene glycol data in Table 12 is

\[ P = -0.54T + 34 \]

where \( P \) is the percentage of propylene glycol in the deicing solution and \( T \) is the temperature at which the solution freezes.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.
(B) Use the model to estimate the freezing temperature to the nearest degree of a solution that is 30% propylene glycol.
(C) Use the model to estimate the percentage of propylene glycol in a solution that freezes at 15°F.

21. **Forestry.** The figure contains a scatter plot of 100 data points for black spruce trees and the linear regression model for this data.

(A) Interpret the slope of the model.
(B) What is the effect of a 1-inch increase in Dbh?
(C) Estimate the height of a black spruce with a Dbh of 12 inches. Round your answer to the nearest foot.
(D) Estimate the Dbh of a black spruce that is 25 feet tall. Round your answer to the nearest inch.

22. **Forestry.** The figure contains a scatter plot of 100 data points for black walnut trees and the linear regression model for this data.

(A) Interpret the slope of the model.
(B) What is the effect of a 1-inch increase in Dbh?
(C) Estimate the height of a black walnut with a Dbh of 12 inches. Round your answer to the nearest foot.
(D) Estimate the Dbh of a black walnut that is 25 feet tall. Round your answer to the nearest inch.

23. **Cable television.** Table 13 shows the increase in both price and revenue for cable television in the United States. The figure shows a scatter plot and a linear regression model for the average monthly price data in Table 13.

(A) Interpret the slope of the model.
(B) Use the model to predict the average monthly price (to the nearest dollar) in 2015.
TABLE 13 Cable Television Price and Revenue

<table>
<thead>
<tr>
<th>Year</th>
<th>Average Monthly Price (dollars)</th>
<th>Annual Total Revenue (billions of dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>16.78</td>
<td>17.58</td>
</tr>
<tr>
<td>1991</td>
<td>18.10</td>
<td>19.43</td>
</tr>
<tr>
<td>1992</td>
<td>19.08</td>
<td>21.08</td>
</tr>
<tr>
<td>1993</td>
<td>19.39</td>
<td>22.84</td>
</tr>
<tr>
<td>1995</td>
<td>23.07</td>
<td>25.42</td>
</tr>
<tr>
<td>1996</td>
<td>24.41</td>
<td>27.71</td>
</tr>
<tr>
<td>1997</td>
<td>26.48</td>
<td>30.49</td>
</tr>
<tr>
<td>1998</td>
<td>27.51</td>
<td>33.50</td>
</tr>
<tr>
<td>1999</td>
<td>28.92</td>
<td>36.92</td>
</tr>
<tr>
<td>2000</td>
<td>30.08</td>
<td>40.86</td>
</tr>
<tr>
<td>2001</td>
<td>31.58</td>
<td>43.52</td>
</tr>
<tr>
<td>2002</td>
<td>34.52</td>
<td>49.43</td>
</tr>
<tr>
<td>2003</td>
<td>36.59</td>
<td>51.30</td>
</tr>
<tr>
<td>2004</td>
<td>38.23</td>
<td>57.60</td>
</tr>
<tr>
<td>2005</td>
<td>39.96</td>
<td>63.09</td>
</tr>
</tbody>
</table>

24. **Cable television.** The figure shows a scatter plot and a linear regression model for the annual revenue data in Table 13.  
(A) Interpret the slope of the model.  
(B) Use the model to predict the annual revenue (to the nearest billion dollars) in 2015.

![Figure for 24](image)

25. **College enrollment.** Table 14 lists the fall enrollment in degree-granting institutions by gender and the figure contains a scatter plot and a regression line for each data set.  
(A) Interpret the slope of each model.  
(B) Predict (to the nearest million) both the male enrollment and the female enrollment in 2010.  
(C) Estimate the first year that female enrollment exceeds male enrollment.

TABLE 14 Fall Enrollment (millions of students)

<table>
<thead>
<tr>
<th>Year</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>5.04</td>
<td>3.54</td>
</tr>
<tr>
<td>1975</td>
<td>6.15</td>
<td>5.04</td>
</tr>
<tr>
<td>1980</td>
<td>5.87</td>
<td>6.22</td>
</tr>
<tr>
<td>1985</td>
<td>5.82</td>
<td>6.43</td>
</tr>
<tr>
<td>1990</td>
<td>6.28</td>
<td>7.53</td>
</tr>
<tr>
<td>1995</td>
<td>6.34</td>
<td>7.92</td>
</tr>
<tr>
<td>2000</td>
<td>6.72</td>
<td>8.59</td>
</tr>
</tbody>
</table>

![Figure for 25; fall enrollment by gender](image)

26. **Graduate school enrollment.** Table 15 lists the fall graduate student enrollment in degree-granting institutions by gender and the figure contains a scatter plot and a regression line for each data set.  
(A) Interpret the slope of each model.  
(B) Predict (to one decimal place) both the male enrollment and the female enrollment in 2010.  
(C) Estimate the first year that female enrollment exceeded male enrollment.

TABLE 15 Fall Graduate School Enrollment (millions of students)

<table>
<thead>
<tr>
<th>Year</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>0.63</td>
<td>0.40</td>
</tr>
<tr>
<td>1975</td>
<td>0.70</td>
<td>0.56</td>
</tr>
<tr>
<td>1980</td>
<td>0.68</td>
<td>0.56</td>
</tr>
<tr>
<td>1985</td>
<td>0.68</td>
<td>0.70</td>
</tr>
<tr>
<td>1990</td>
<td>0.74</td>
<td>0.85</td>
</tr>
<tr>
<td>1995</td>
<td>0.77</td>
<td>0.96</td>
</tr>
<tr>
<td>2000</td>
<td>0.78</td>
<td>1.07</td>
</tr>
</tbody>
</table>

![Figure for 26; fall enrollment by gender](image)
Problems 27–30 require a graphing calculator or a computer that can calculate the linear regression line for a given data set.

27. Olympic Games. Find a linear regression model for the men’s 100-meter freestyle data given in Table 16, where \( x \) is years since 1968 and \( y \) is winning time (in seconds). Do the same for the women’s 100-meter freestyle data. (Round regression coefficients to three decimal places.) Do these models indicate that the women will eventually catch up with the men? If so, when? Do you think this will actually occur?

28. Olympic Games. Find a linear regression model for the men’s 200-meter backstroke data given in Table 16, where \( x \) is years since 1968 and \( y \) is winning time (in seconds). Do the same for the women’s 200-meter backstroke data. (Round regression coefficients to four decimal places.) Do these models indicate that the women will eventually catch up with the men? If so, when? Do you think this will actually occur?

29. Supply and demand. Table 17 contains price–supply data and price–demand data for corn. Find a linear regression model for the price–supply data where \( x \) is supply (in billions of bushels) and \( y \) is price (in dollars). Do the same for the price–demand data. (Round regression coefficients to two decimal places.) Find the equilibrium price for corn.

30. Supply and demand. Table 18 contains price–supply data and price–demand data for soybeans. Find a linear regression model for the price–supply data where \( x \) is supply (in billions of bushels) and \( y \) is price (in dollars). Do the same for the price–demand data. (Round regression coefficients to two decimal places.) Find the equilibrium price for soybeans.
A suggested strategy (p. 8-9) can be used to solve many word problems.

- A company breaks even if revenues costs C, makes a profit if R > C, and incurs a loss if R < C.

1-2 Graphs and Lines

- A Cartesian or rectangular coordinate system is formed by the intersection of a horizontal real number line, usually called the x axis, and a vertical real number line, usually called the y axis, at their origins. The axes determine a plane and divide this plane into four quadrants. Each point in the plane corresponds to its coordinates—an ordered pair (a, b) determined by passing horizontal and vertical lines through the point. The abscissa or x coordinate a is the coordinate of the intersection of the vertical line and the x axis, and the ordinate or y coordinate b is the coordinate of the intersection of the horizontal line and the y axis. The point with coordinates (0, 0) is called the origin.

- The standard form for a linear equation in two variables is Ax + By = C, with A and B not both zero. The graph of this equation is a line, and every line in a Cartesian coordinate system is the graph of a linear equation.

- The graph of the equation x = a is a vertical line and the graph of y = b is a horizontal line.

- If (x₁, y₁) and (x₂, y₂) are two distinct points on a line, then m = (y₂ - y₁)/(x₂ - x₁) is the slope of the line.

- The equation y = mx + b is the slope-intercept form of the equation of the line with slope m and y intercept b.

- The point-slope form of the equation of the line with slope m that passes through (x₁, y₁) is y - y₁ = m(x - x₁).

- In a competitive market, the intersection of the supply equation and the demand equation is called the equilibrium point, the corresponding price is called the equilibrium price, and the common value of supply and demand is called the equilibrium quantity.

1-3 Linear Regression

- If the variables x and y are related by the equation y = mx + b, then x and y are linearly related and the slope m is the rate of change of y with respect to x.

- Regression analysis is used to fit a curve to a data set, usually with the aid of a graphing calculator or a computer. A graph of the points in a data set is called a scatter plot. A regression model can be used to interpolate between points in a data set or to extrapolate or predict points outside the data set.

**REVIEW EXERCISE**

Work through all the problems in this chapter review and check answers in the back of the book. Answers to all review problems are there, and following each answer is a number in italics indicating the section in which that type of problem is discussed. Where weaknesses show up, review appropriate sections in the text.

A

1. Solve 2x + 3 = 7x - 11.
2. Solve \( \frac{x}{12} - \frac{x - 3}{3} = \frac{1}{2} \).
3. Solve 2x + 5y = 9 for y.
4. Solve 3x - 4y = 7 for x.

Solve Problems 5–7 and graph on a real number line.

5. \( 4y - 3 < 10 \)
6. \( -1 < -2x + 5 \leq 3 \)
7. \( 1 - \frac{x - 3}{3} \leq \frac{1}{2} \)
8. Sketch a graph of 3x + 2y = 9.

9. Write an equation of a line with x intercept 6 and y intercept 4. Write the final answer in the form \( Ax + By = C \).
10. Sketch a graph of \( 2x - 3y = 18 \). What are the intercepts and slope of the line?
11. Write an equation in the form \( y = mx + b \) for a line with slope \( \frac{2}{3} \) and y intercept 6.
12. Write the equations of the vertical line and the horizontal line that pass through \((-6, 5)\).
13. Write the equation of a line through each indicated point with the indicated slope. Write the final answer in the form \( y = mx + b \).
   (A) \( m = \frac{2}{3}; \) \((-3, 2)\)  \( \text{(B)} \) \( m = 0; \) \((3, 3)\)
14. Write the equation of the line through the two indicated points. Write the final answer in the form \(Ax + By = C\).
(A) \((-3, 5), (1, -1)\)  (B) \((-1, 5), (4, 5)\)
(C) \((-2, 7), (-2, -2)\)

15. 3x + 25 = 5x
16. \(\frac{u}{5} = \frac{u + 6}{5}\)
17. \(\frac{5x}{3} - \frac{4 + x}{2} = \frac{x - 2}{4} + 1\)
18. \(0.05x + 0.25(30 - x) = 3.3\)
19. \(0.2(x - 3) + 0.05x = 0.4\)

Solve Problems 20–24 and graph on a real number line.

20. \(2x + 4 > 5x - 4\)
21. \(3(2 - x) - 2 \leq 2x - 1\)
22. \(\frac{x + 3}{8} - \frac{4 + x}{2} > 5 - \frac{2 - x}{3}\)
23. \(-5 \leq 3 - 2x < 1\)
24. \(-1.5 \leq 2 - 4x \leq 0.5\)

25. Given \(Ax + By = 30\), graph each of the following cases on the same coordinate axes.
(A) \(A = 5\) and \(B = 0\)
(B) \(A = 0\) and \(B = 6\)
(C) \(A = 6\) and \(B = 5\)

26. Describe the graphs of \(x = -3\) and \(y = 2\). Graph both simultaneously in the same coordinate system.

27. Describe the lines defined by the following equations:
(A) \(3x + 4y = 0\)  (B) \(3x + 4 = 0\)
(C) \(4y = 0\)  (D) \(3x + 4y - 36 = 0\)

28. \(A = \frac{1}{2}(a + bh); \) for \(a(h \neq 0)\)
29. \(S = \frac{1}{2} - \frac{P}{dt}\) for \(d(dt \neq 1)\)
30. For what values of \(a\) and \(b\) is the inequality \(a + b < b - a\) true?
31. If \(a\) and \(b\) are negative numbers and \(a > b\), then is \(ab\) greater than 1 or less than 1?
32. Graph \(y = mx + b\) and \(y = -\frac{1}{m} + b\) simultaneously in the same coordinate system for \(b\) fixed and several different values of \(m, m \neq 0\). Describe the apparent relationship between the graphs of the two equations.

APPLIEDS

33. An investor has \(\$300,000\) to invest. If part is invested at 5% and the rest at 9%, how much should be invested at 5% to yield 8% on the total amount?

34. Break-even analysis. A producer of educational CDs is producing an instructional CD. The producer estimates that it will cost \(\$90,000\) to record the CD and \(\$5.10\) per unit to copy and distribute the CD. If the wholesale price of the CD is \(\$14.70\), how many CDs must be sold for the producer to break even?

35. Sports medicine. A simple rule of thumb for determining your maximum safe heart rate (in beats per minute) is to subtract your age from 220. When exercising, you should maintain a heart rate between 60% and 85% of your maximum safe rate.
(A) Find a linear model for the minimum heart rate \(m\) for a person of age \(x\) years.
(B) Find a linear model for the maximum heart rate \(M\) for a person of age \(x\) years.
(C) What range of heartbeats should you maintain while exercising if you are 20 years old?
(D) What range of heartbeats should you maintain while exercising if you are 50 years old?

36. Linear depreciation. A bulldozer was purchased by a construction company for \(\$224,000\) and is assumed to have a depreciated value of \(\$100,000\) after 8 years. If the value is depreciated linearly from \(\$224,000\) to \(\$100,000\),
(A) Find the linear equation that relates value \(V\) (in dollars) to time \(t\) (in years).
(B) What would be the depreciated value of the system after 12 years?

37. Business—Pricing. A sporting goods store sells tennis rackets that cost \(\$130\) for \(\$208\) and court shoes that cost \(\$50\) for \(\$80\).
(A) If the markup policy of the store for items that cost over \(\$10\) is assumed to be linear and is reflected in the pricing of these two items, write an equation that expresses retail price \(R\) in terms of cost \(C\).
(B) What would be the retail price of a pair of in-line skates that cost \(\$120\)?
(C) What would be the cost of a pair of cross-country skis that had a retail price of \(\$176\)?
(D) What is the slope of the graph of the equation found in part (A)? Interpret the slope relative to the problem.

38. Income. A salesperson receives a base salary of \(\$200\) per week and a commission of 10% on all sales over \(\$3,000\) during the week. Find the weekly earnings for weekly sales of \(\$2,000\) and for weekly sales of \(\$5,000\).

39. Price-demand. The weekly demand for mouthwash in a chain of drug stores is 1,160 bottles at a price of \(\$3.79\) each. If the price is lowered to \(\$3.29\), the weekly demand increases to 1,320 bottles. Assuming that the relationship between the weekly demand \(x\) and the price per bottle \(p\) is linear, express \(p\) in terms of \(x\). How many bottles would the stores sell each week if the price were lowered to \(\$3.29\)?
40. **Freezing temperature.** Methanol, also known as wood alcohol, can be used as a fuel for suitably equipped vehicles. It was also widely used as an antifreeze in cars until the late 1930s and is still used today in special applications, such as pipelines. Table 1 lists the freezing temperature for various concentrations (as a percentage of total weight) of methanol in water. A linear regression model for the data in Table 1 is

\[ T = 40 - 2M \]

where $M$ is the percentage of methanol in the solution and $T$ is the temperature at which the solution freezes.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Methanol (%Wt)</th>
<th>Freezing temperature (°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>−15</td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>−40</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>−65</td>
</tr>
<tr>
<td>60</td>
<td>60</td>
<td>−95</td>
</tr>
</tbody>
</table>

Source: Ashland Inc.

(A) Draw a scatter plot of the data and a graph of the model on the same axes.

(B) Use the model to estimate the freezing temperature to the nearest degree of a solution that is 35% methanol.

(C) Use the model to estimate the percentage of methanol in a solution that freezes at −50°F.

42. **Consumer Price Index.** The United States Consumer Price Index (CPI) in recent years is given in Table 3. A scatter plot of the data and the linear regression line are shown in the figure. (A) Interpret the slope of the model. (B) Predict the CPI in 2010.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Consumer Price Index (1982–1984 = 100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>CPI</td>
</tr>
<tr>
<td>1990</td>
<td>130.7</td>
</tr>
<tr>
<td>1991</td>
<td>136.2</td>
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<tr>
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<td>144.5</td>
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<td>177.1</td>
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<tr>
<td>2002</td>
<td>179.9</td>
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<tr>
<td>2003</td>
<td>184.0</td>
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<tr>
<td>2004</td>
<td>188.9</td>
</tr>
<tr>
<td>2005</td>
<td>195.3</td>
</tr>
</tbody>
</table>

Source: U.S. Bureau of Labor Statistics

43. **Forestry.** The figure contains a scatter plot of 20 data points for white pine trees and the linear regression model for this data. (A) Interpret the slope of the model. (B) What is the effect of a 1-inch increase in Dbh? (C) Estimate the height of a white pine tree with a Dbh of 25 inches. Round your answer to the nearest foot. (D) Estimate the Dbh of a white pine tree that is 15 feet tall. Round your answer to the nearest inch.

(A) Interpret the slope of each model.

(B) Draw a scatter plot of both data sets and a graph of both models in the same coordinate systems.

(C) In which year will time spent by women surpass time spent by men?