The objective of Chapter 2 is to analyze short-term alternatives when the time value of money is not a factor.

The A380 Superjumbo’s Breakeven Point

When Europe’s Airbus Company approved the A380 program in 2000, it was estimated that only 250 of the giant, 555-seat aircraft needed to be sold to breakeven. The program was initially based on expected deliveries of 751 aircraft over its life cycle. Long delays and mounting costs however, have dramatically changed the original breakeven figure. In 2005, this figure was updated to 270 aircraft. According to an article in the Financial Times (October 20, 2006, p. 18), Airbus would have to sell 420 aircraft to breakeven—a 68% increase over the original estimate. To date, only 159 firm orders for the aircraft have been received. The topic of breakeven analysis is an integral part of this chapter.
The correct solution to any problem depends primarily on a true understanding of what the problem really is.

—Arthur M. Wellington (1887)

2.1 Cost Terminology

There are a variety of costs to be considered in an engineering economic analysis.* These costs differ in their frequency of occurrence, relative magnitude, and degree of impact on the study. In this section, we define a number of cost categories and illustrate how they should be treated in an engineering economic analysis.

2.1.1 Fixed, Variable, and Incremental Costs

*For the purposes of this book, the words cost and expense are used interchangeably.

Fixed costs are those unaffected by changes in activity level over a feasible range of operations for the capacity or capability available. Typical fixed costs include insurance and taxes on facilities, general management and administrative salaries, license fees, and interest costs on borrowed capital.

Of course, any cost is subject to change, but fixed costs tend to remain constant over a specific range of operating conditions. When larger changes in usage of resources occur, or when plant expansion or shutdown is involved, fixed costs can be affected.

Variable costs are those associated with an operation that vary in total with the quantity of output or other measures of activity level. For example, the costs of material and labor used in a product or service are variable costs, because they vary in total with the number of output units, even though the costs per unit stay the same.

An incremental cost (or incremental revenue) is the additional cost (or revenue) that results from increasing the output of a system by one (or more) units. Incremental cost is often associated with “go–no go” decisions that involve a limited change in output or activity level. For instance, the incremental cost per mile for driving an automobile may be $0.49, but this cost depends on considerations such as total mileage driven during the year (normal operating range), mileage expected for the next major trip, and the age of the automobile. Also, it is common to read about the “incremental cost of producing a barrel of oil” and “incremental cost to the state for educating a student.” As these examples indicate, the incremental cost (or revenue) is often quite difficult to determine in practice.

**EXAMPLE 2-1 Fixed and Variable Costs**

In connection with surfacing a new highway, a contractor has a choice of two sites on which to set up the asphalt-mixing plant equipment. The contractor estimates that it will cost $1.15 per cubic yard per mile (yd³/mile) to haul the asphalt-paving material from the mixing plant to the job location. Factors relating to the two mixing sites are as follows (production costs at each site are the same):
The job requires 50,000 cubic yards of mixed-asphalt-paving material. It is estimated that four months (17 weeks of five working days per week) will be required for the job. Compare the two sites in terms of their fixed, variable, and total costs. Assume that the cost of the return trip is negligible. Which is the better site? For the selected site, how many cubic yards of paving material does the contractor have to deliver before starting to make a profit if paid $8.05 per cubic yard delivered to the job location?

Solution

The fixed and variable costs for this job are indicated in the table shown next. Site rental, setup, and removal costs (and the cost of the flagperson at Site B) would be constant for the total job, but the hauling cost would vary in total amount with the distance and thus with the total output quantity of yd^3-miles.

<table>
<thead>
<tr>
<th>Cost Factor</th>
<th>Site A</th>
<th>Site B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average hauling distance</td>
<td>6 miles</td>
<td>4.3 miles</td>
</tr>
<tr>
<td>Monthly rental of site</td>
<td>$1,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>Cost to set up and remove equipment</td>
<td>$15,000</td>
<td>$25,000</td>
</tr>
<tr>
<td>Hauling expense</td>
<td>$1.15/yd^3-mile</td>
<td>$1.15/yd^3-mile</td>
</tr>
<tr>
<td>Flagperson</td>
<td>Not required</td>
<td>$96/day</td>
</tr>
</tbody>
</table>

Site B, which has the larger fixed costs, has the smaller total cost for the job. Note that the extra fixed costs of Site B are being “traded off” for reduced variable costs at this site.

The contractor will begin to make a profit at the point where total revenue equals total cost as a function of the cubic yards of asphalt pavement mix delivered. Based on Site B, we have

\[
4.3(1.15) = 4.945 \text{ in variable cost per yd}^3 \text{ delivered}
\]

Total cost = total revenue

\[
53,160 + 4.945x = 8.05x
\]

\[
x = 17,121 \text{ yd}^3 \text{ delivered.}
\]

Therefore, by using Site B, the contractor will begin to make a profit on the job after delivering 17,121 cubic yards of material.
2.1.2 Direct, Indirect, and Standard Costs

These frequently encountered cost terms involve most of the cost elements that also fit into the previous overlapping categories of fixed and variable costs and recurring and nonrecurring costs. Direct costs are costs that can be reasonably measured and allocated to a specific output or work activity. The labor and material costs directly associated with a product, service, or construction activity are direct costs. For example, the materials needed to make a pair of scissors would be a direct cost.

Indirect costs are costs that are difficult to attribute or allocate to a specific output or work activity. Normally, they are costs allocated through a selected formula (such as proportional to direct labor hours, direct labor dollars, or direct material dollars) to the outputs or work activities. For example, the costs of common tools, general supplies, and equipment maintenance in a plant are treated as indirect costs.

Overhead consists of plant operating costs that are not direct labor or direct material costs. In this book, the terms indirect costs, overhead, and burden are used interchangeably. Examples of overhead include electricity, general repairs, property taxes, and supervision. Administrative and selling expenses are usually added to direct costs and overhead costs to arrive at a unit selling price for a product or service. (Appendix 2-A provides a more detailed discussion of cost accounting principles.)

Standard costs are planned costs per unit of output that are established in advance of actual production or service delivery. They are developed from anticipated direct labor hours, materials, and overhead categories (with their established costs per unit). Because total overhead costs are associated with a certain level of production, this is an important condition that should be remembered when dealing with standard cost data (for example, see Section 2.4.3). Standard costs play an important role in cost control and other management functions. Some typical uses are the following:

1. Estimating future manufacturing costs
2. Measuring operating performance by comparing actual cost per unit with the standard unit cost
3. Preparing bids on products or services requested by customers
4. Establishing the value of work in process and finished inventories

2.1.3 Cash Cost versus Book Cost

A cost that involves payment of cash is called a cash cost (and results in a cash flow) to distinguish it from one that does not involve a cash transaction and is reflected in the accounting system as a noncash cost. This noncash cost is often referred to as a book cost. Cash costs are estimated from the perspective established for the analysis (Principle 3, Section 1.2) and are the future expenses incurred for the alternatives being analyzed. Book costs are costs that do not involve cash payments but rather represent the recovery of past expenditures over a fixed period of time. The most common example of book cost is the depreciation charged for the use of assets such as plant and equipment. In engineering economic analysis, only those costs that are cash flows or potential cash flows from the defined perspective for the analysis need to be considered. Depreciation, for example, is not a cash flow and is important in
an analysis only because it affects income taxes, which are cash flows. We discuss the topics of depreciation and income taxes in Chapter 7.

### 2.1.4 Sunk Cost

A *sunk cost* is one that has occurred in the past and has no relevance to estimates of future costs and revenues related to an alternative course of action. Thus, a sunk cost is common to all alternatives, is not part of the future (prospective) cash flows, and can be disregarded in an engineering economic analysis. For instance, sunk costs are nonrefundable cash outlays, such as earnest money on a house or money spent on a passport.

The concept of sunk cost is illustrated in the next simple example. Suppose that Joe College finds a motorcycle he likes and pays $40 as a down payment, which will be applied to the $1,300 purchase price, but which must be forfeited if he decides not to take the cycle. Over the weekend, Joe finds another motorcycle he considers equally desirable for a purchase price of $1,230. For the purpose of deciding which cycle to purchase, the $40 is a sunk cost and thus would not enter into the decision, except that it lowers the remaining cost of the first cycle. The decision then is between paying an additional $1,260 ($1,300 − $40) for the first motorcycle versus $1,230 for the second motorcycle.

In summary, sunk costs are irretrievable consequences of past decisions and therefore are irrelevant in the analysis and comparison of alternatives that affect the future. Even though it is sometimes emotionally difficult to do, sunk costs should be ignored, except possibly to the extent that their existence assists you to anticipate better what will happen in the future.

#### EXAMPLE 2-2 Sunk Costs in Replacement Analysis

A classic example of sunk cost involves the replacement of assets. Suppose that your firm is considering the replacement of a piece of equipment. It originally cost $50,000, is presently shown on the company records with a value of $20,000, and can be sold for an estimated $5,000. For purposes of replacement analysis, the $50,000 is a sunk cost. However, one view is that the sunk cost should be considered as the difference between the value shown in the company records and the present realizable selling price. According to this viewpoint, the sunk cost is $20,000 minus $5,000, or $15,000. Neither the $50,000 nor the $15,000, however, should be considered in an engineering economic analysis, except for the manner in which the $15,000 may affect income taxes, which will be discussed in Chapter 9.

### 2.1.5 Opportunity Cost

An *opportunity cost* is incurred because of the use of limited resources, such that the opportunity to use those resources to monetary advantage in an alternative use is foregone. Thus, it is the cost of the best rejected (i.e., foregone) opportunity and is often hidden or implied.
Consider a student who could earn $20,000 for working during a year, but chooses instead to go to school for a year and spend $5,000 to do so. The opportunity cost of going to school for that year is $25,000: $5,000 cash outlay and $20,000 for income foregone. (This figure neglects the influence of income taxes and assumes that the student has no earning capability while in school.)

### EXAMPLE 2-3  
**Opportunity Cost in Replacement Analysis**

The concept of an opportunity cost is often encountered in analyzing the replacement of a piece of equipment or other capital asset. Let us reconsider Example 2-2, in which your firm considered the replacement of an existing piece of equipment that originally cost $50,000, is presently shown on the company records with a value of $20,000, but has a present market value of only $5,000. For purposes of an engineering economic analysis of whether to replace the equipment, the present investment in that equipment should be considered as $5,000, because, by keeping the equipment, the firm is giving up the opportunity to obtain $5,000 from its disposal. Thus, the $5,000 immediate selling price is really the investment cost of not replacing the equipment and is based on the opportunity cost concept.

### 2.1.6 Life-Cycle Cost

In engineering practice, the term *life-cycle cost* is often encountered. This term refers to a summation of all the costs related to a product, structure, system, or service during its life span. The *life cycle* is illustrated in Figure 2-1. The life cycle begins with identification of the economic need or want (the requirement) and ends with retirement and disposal activities. It is a time horizon that must be defined in the context of the specific situation—whether it is a highway bridge, a jet engine for commercial aircraft, or an automated flexible manufacturing cell for a factory. The end of the life cycle may be projected on a functional or an economic basis. For example, the amount of time that a structure or piece of equipment is able to perform economically may be shorter than that permitted by its physical capability. Changes in the design efficiency of a boiler illustrate this situation. The old boiler may be able to produce the steam required, but not economically enough for the intended use.

The life cycle may be divided into two general time periods: the acquisition phase and the operation phase. As shown in Figure 2-1, each of these phases is further subdivided into interrelated but different activity periods.

The acquisition phase begins with an analysis of the economic need or want—the analysis necessary to make explicit the requirement for the product, structure, system, or service. Then, with the requirement explicitly defined, the other activities in the acquisition phase can proceed in a logical sequence. The conceptual design activities translate the defined technical and operational requirements into a preferred preliminary design. Included in these activities are development of the feasible alternatives and engineering economic analyses to assist in selection of the preferred preliminary design. Also, advanced development
and prototype-testing activities to support the preliminary design work occur during this period.

The next group of activities in the acquisition phase involves detailed design and planning for production or construction. This step is followed by the activities necessary to prepare, acquire, and make ready for operation the facilities and other resources needed. Again, engineering economy studies are an essential part of the design process to analyze and compare alternatives and to assist in determining the final detailed design.

In the operation phase, the production, delivery, or construction of the end item(s) or service and their operation or customer use occur. This phase ends with retirement from active operation or use and, often, disposal of the physical assets involved. The priorities for engineering economy studies during the operation phase are (1) achieving efficient and effective support to operations, (2) determining whether (and when) replacement of assets should occur, and (3) projecting the timing of retirement and disposal activities.

Figure 2-1 shows relative cost profiles for the life cycle. The greatest potential for achieving life-cycle cost savings is early in the acquisition phase. How much of
the life-cycle costs for a product (for example) can be saved is dependent on many factors. However, effective engineering design and economic analysis during this phase are critical in maximizing potential savings.

The cumulative committed life-cycle cost curve increases rapidly during the acquisition phase. In general, approximately 80% of life-cycle costs are “locked in” at the end of this phase by the decisions made during requirements analysis and preliminary and detailed design. In contrast, as reflected by the cumulative life-cycle cost curve, only about 20% of actual costs occur during the acquisition phase, with about 80% being incurred during the operation phase.

Thus, one purpose of the life-cycle concept is to make explicit the interrelated effects of costs over the total life span for a product. An objective of the design process is to minimize the life-cycle cost—while meeting other performance requirements—by making the right trade-offs between prospective costs during the acquisition phase and those during the operation phase.

The cost elements of the life cycle that need to be considered will vary with the situation. Because of their common use, however, several basic life-cycle cost categories will now be defined.

The **investment cost** is the capital required for most of the activities in the acquisition phase. In simple cases, such as acquiring specific equipment, an investment cost may be incurred as a single expenditure. On a large, complex construction project, however, a series of expenditures over an extended period could be incurred. This cost is also called a **capital investment**.

The term **working capital** refers to the funds required for current assets (i.e., other than fixed assets such as equipment, facilities, etc.) that are needed for the start-up and support of operational activities. For example, products cannot be made or services delivered without having materials available in inventory. Functions such as maintenance cannot be supported without spare parts, tools, trained personnel, and other resources. Also, cash must be available to pay employee salaries and the other expenses of operation. The amount of working capital needed will vary with the project involved, and some or all of the investment in working capital is usually recovered at the end of a project’s life.

**Operation and maintenance cost** (O&M) includes many of the recurring annual expense items associated with the operation phase of the life cycle. The direct and indirect costs of operation associated with the five primary resource areas—people, machines, materials, energy, and information—are a major part of the costs in this category.

**Disposal cost** includes those nonrecurring costs of shutting down the operation and the retirement and disposal of assets at the end of the life cycle. Normally, costs associated with personnel, materials, transportation, and one-time special activities can be expected. These costs will be offset in some instances by receipts from the sale of assets with remaining market value. A classic example of a disposal cost is that associated with cleaning up a site where a chemical processing plant had been located.
2.2 The General Economic Environment

There are numerous general economic concepts that must be taken into account in engineering studies. In broad terms, economics deals with the interactions between people and wealth, and engineering is concerned with the cost-effective use of scientific knowledge to benefit humankind. This section introduces some of these basic economic concepts and indicates how they may be factors for consideration in engineering studies and managerial decisions.

2.2.1 Consumer and Producer Goods and Services

The goods and services that are produced and utilized may be divided conveniently into two classes. Consumer goods and services are those products or services that are directly used by people to satisfy their wants. Food, clothing, homes, cars, television sets, haircuts, opera, and medical services are examples. The providers of consumer goods and services must be aware of, and are subject to, the changing wants of the people to whom their products are sold.

Producer goods and services are used to produce consumer goods and services or other producer goods. Machine tools, factory buildings, buses, and farm machinery are examples. The amount of producer goods needed is determined indirectly by the amount of consumer goods or services that are demanded by people. However, because the relationship is much less direct than for consumer goods and services, the demand for and production of producer goods may greatly precede or lag behind the demand for the consumer goods that they will produce.

2.2.2 Measures of Economic Worth

Goods and services are produced and desired because they have utility—the power to satisfy human wants and needs. Thus, they may be used or consumed directly, or they may be used to produce other goods or services. Utility is most commonly measured in terms of value, expressed in some medium of exchange as the price that must be paid to obtain the particular item.

Much of our business activity, including engineering, focuses on increasing the utility (value) of materials and products by changing their form or location. Thus, iron ore, worth only a few dollars per ton, significantly increases in value by being processed, combined with suitable alloying elements, and converted into razor blades. Similarly, snow, worth almost nothing when high in distant mountains, becomes quite valuable when it is delivered in melted form several hundred miles away to dry southern California.

2.2.3 Necessities, Luxuries, and Price Demand

Goods and services may be divided into two types: necessities and luxuries. Obviously, these terms are relative, because, for most goods and services, what one person considers a necessity may be considered a luxury by another. For example, a person living in one community may find that an automobile is a necessity to get
CHAPTER 2 / COST CONCEPTS AND DESIGN ECONOMICS

Figure 2.2 General Price–Demand Relationship. (Note that price is considered to be the independent variable but is shown as the vertical axis. This convention is commonly used by economists.)

![Price-Demand Relationship](image)

Figure 2.2 General Price–Demand Relationship. (Note that price is considered to be the independent variable but is shown as the vertical axis. This convention is commonly used by economists.)

To and from work. If the same person lived and worked in a different city, adequate public transportation might be available, and an automobile would be a luxury. For all goods and services, there is a relationship between the price that must be paid and the quantity that will be demanded or purchased. This general relationship is depicted in Figure 2-2. As the selling price per unit \( p \) is increased, there will be less demand \( D \) for the product, and as the selling price is decreased, the demand will increase. The relationship between price and demand can be expressed as the linear function

\[
p = a - bD 
\]

for \( 0 \leq D \leq \frac{a}{b} \), and \( a > 0, b > 0 \), \( (2-1) \)

where \( a \) is the intercept on the price axis and \( -b \) is the slope. Thus, \( b \) is the amount by which demand increases for each unit decrease in \( p \). Both \( a \) and \( b \) are constants.

It follows, of course, that

\[
D = \frac{a - p}{b} \quad (b \neq 0). \quad (2-2)
\]

2.2.4 Competition

Because economic laws are general statements regarding the interaction of people and wealth, they are affected by the economic environment in which people and wealth exist. Most general economic principles are stated for situations in which perfect competition exists.

Perfect competition occurs in a situation in which any given product is supplied by a large number of vendors and there is no restriction on additional suppliers entering the market. Under such conditions, there is assurance of complete freedom on the part of both buyer and seller. Perfect competition may never occur in actual practice, because of a multitude of factors that impose some degree of limitation...
upon the actions of buyers or sellers, or both. However, with conditions of perfect competition assumed, it is easier to formulate general economic laws.

Monopoly is at the opposite pole from perfect competition. A perfect monopoly exists when a unique product or service is only available from a single supplier and that vendor can prevent the entry of all others into the market. Under such conditions, the buyer is at the complete mercy of the supplier in terms of the availability and price of the product. Perfect monopolies rarely occur in practice, because (1) few products are so unique that substitutes cannot be used satisfactorily and (2) governmental regulations prohibit monopolies if they are unduly restrictive.

2.2.5 The Total Revenue Function

The total revenue, TR, that will result from a business venture during a given period is the product of the selling price per unit, $p$, and the number of units sold, $D$. Thus,

$$TR = price \times demand = p \cdot D.$$  \hspace{1cm} (2-3)

If the relationship between price and demand as given in Equation (2-1) is used,

$$TR = (a - bD)D = aD - bD^2 \quad \text{for } 0 \leq D \leq \frac{a}{b} \text{ and } a > 0, b > 0.$$  \hspace{1cm} (2-4)

The relationship between total revenue and demand for the condition expressed in Equation (2-4) may be represented by the curve shown in Figure 2-3. From calculus, the demand, $\hat{D}$, that will produce maximum total revenue can be obtained by solving

$$\frac{dTR}{dD} = a - 2bD = 0.$$  \hspace{1cm} (2-5)
Thus,*

\[ \hat{D} = \frac{a}{2b}. \]  

(2-6)

It must be emphasized that, because of cost–volume relationships (discussed in the next section), most businesses would not obtain maximum profits by maximizing revenue. Accordingly, the cost–volume relationship must be considered and related to revenue, because cost reductions provide a key motivation for many engineering process improvements.

### 2.2.6 Cost, Volume, and Breakeven Point Relationships

Fixed costs remain constant over a wide range of activities, but variable costs vary in total with the volume of output (Section 2.1.1). Thus, at any demand \( D \), total cost is

\[ C_T = C_F + C_V, \]  

(2-7)

where \( C_F \) and \( C_V \) denote fixed and variable costs, respectively. For the linear relationship assumed here,

\[ C_V = c_v \cdot D, \]  

(2-8)

where \( c_v \) is the variable cost per unit. In this section, we consider two scenarios for finding breakeven points. In the first scenario, demand is a function of price. The second scenario assumes that price and demand are independent of each other.

**Scenario 1** When total revenue, as depicted in Figure 2-3, and total cost, as given by Equations (2-7) and (2-8), are combined, the typical results as a function of demand are depicted in Figure 2-4. At breakeven point \( D_1^* \), total revenue is equal

---

* To guarantee that \( \hat{D} \) maximizes total revenue, check the second derivative to be sure it is negative:

\[ \frac{d^2 TR}{dD^2} = -2b. \]

Also, recall that in cost-minimization problems a positively signed second derivative is necessary to guarantee a minimum-value optimal cost solution.
to total cost, and an increase in demand will result in a profit for the operation. Then at optimal demand, $D^*$, profit is maximized [Equation (2-10)]. At breakeven point $D'_b$, total revenue and total cost are again equal, but additional volume will result in an operating loss instead of a profit. Obviously, the conditions for which breakeven and maximum profit occur are our primary interest. First, at any volume (demand), $D$,

\[
\text{Profit (loss)} = \text{total revenue} - \text{total costs} = (aD - bD^2) - (C_F + c_v D) \\
= -bD^2 + (a - c_v)D - C_F \quad \text{for } 0 \leq D \leq \frac{a}{b} \text{ and } a > 0, \ b > 0. \quad (2-9)
\]

In order for a profit to occur, based on Equation (2-9), and to achieve the typical results depicted in Figure 2-4, two conditions must be met:

1. $(a - c_v) > 0$; that is, the price per unit that will result in no demand has to be greater than the variable cost per unit. (This avoids negative demand.)

2. $\text{TR} > \text{total cost } (C_T)$ for the period involved.

If these conditions are met, we can find the optimal demand at which maximum profit will occur by taking the first derivative of Equation (2-9) with respect to $D$ and setting it equal to zero:

\[
\frac{d(\text{profit})}{dD} = a - c_v - 2bD = 0.
\]

The optimal value of $D$ that maximizes profit is

\[
D^* = \frac{a - c_v}{2b}. \quad (2-10)
\]

To ensure that we have maximized profit (rather than minimized it), the sign of the second derivative must be negative. Checking this, we find that

\[
\frac{d^2(\text{profit})}{dD^2} = -2b,
\]

which will be negative for $b > 0$ (as earlier specified).

An economic breakeven point for an operation occurs when total revenue equals total cost. Then for total revenue and total cost, as used in the development of Equations (2-9) and (2-10) and at any demand $D$,

\[
\text{Total revenue} = \text{total cost} \quad \text{(breakeven point)} \\
aD - bD^2 = C_F + c_v D \\
-bD^2 + (a - c_v)D - C_F = 0. \quad (2-11)
\]
Because Equation (2-11) is a quadratic equation with one unknown \((D)\), we can solve for the breakeven points \(D'_1\) and \(D'_2\) (the roots of the equation):\(^{a}\)

\[
D' = \frac{-(a - cv) \pm \sqrt{(a - cv)^2 - 4(-b)(-CF)}}{2(-b)}. \quad (2-12)
\]

With the conditions for a profit satisfied [Equation (2-9)], the quantity in the brackets of the numerator (the discriminant) in Equation (2-12) will be greater than zero. This will ensure that \(D'_1\) and \(D'_2\) have real positive, unequal values.

**EXAMPLE 2-4  Optimal Demand When Demand Is a Function of Price**

A company produces an electronic timing switch that is used in consumer and commercial products. The fixed cost (\(C_F\)) is $73,000 per month, and the variable cost (\(cv\)) is $83 per unit. The selling price per unit is \(p = $180 - 0.02(D)\), based on Equation (2-1). For this situation,

(a) determine the optimal volume for this product and confirm that a profit occurs (instead of a loss) at this demand.

(b) find the volumes at which breakeven occurs; that is, what is the range of profitable demand? Solve by hand and by spreadsheet.

**Solution by Hand**

(a) \(D^* = \frac{a - cv}{2b} = \frac{$180 - $83}{2(0.02)} = 2,425\) units per month [from Equation (2-10)].

Is \((a - cv) > 0\)?

\(($180 - $83) = $97\), which is greater than 0.

And is \((\text{total revenue} - \text{total cost}) > 0\) for \(D^* = 2,425\) units per month?

\[
[$180(2,425) - 0.02(2,425)^2] - [$73,000 + $83(2,425)] = $44,612
\]

A demand of \(D^* = 2,425\) units per month results in a maximum profit of $44,612 per month. Notice that the second derivative is negative \((-0.04)\).

(b) Total revenue = total cost (breakeven point)

\[-bD^2 + (a - cv)D - C_F = 0 \quad \text{[from Equation (2-11)]}\]

\[
-0.02D^2 + ($180 - $83)D - $73,000 = 0
\]

\[
-0.02D^2 + 97D - 73,000 = 0
\]

---

* Given the quadratic equation \(ax^2 + bx + c = 0\), the roots are given by \(x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\).
And, from Equation (2-12),

\[
D' = \frac{-97 \pm [(97)^2 - 4(-0.02)(-73,000)]^{0.5}}{2(-0.02)}
\]

\[
D'_1 = \frac{-97 + 59.74}{-0.04} = 932 \text{ units per month}
\]

\[
D'_2 = \frac{-97 - 59.74}{-0.04} = 3,918 \text{ units per month.}
\]

Thus, the range of profitable demand is 932–3,918 units per month.

**Spreadsheet Solution**

Figure 2-5(a) displays the spreadsheet solution for this problem. This spreadsheet calculates profit for a range of demand values (shown in column A). For a specific value of demand, price per unit is calculated in column B by using Equation (2-1) and Total Revenue is simply demand \times price. Total Expense is computed by using Equations (2-7) and (2-8). Finally, Profit (column E) is then Total Revenue – Total Expense.

A quick inspection of the Profit column gives us an idea of the optimal demand value as well as the breakeven points. Note that profit steadily increases as demand increases to 2,500 units per month and then begins to drop off. This tells us that the optimal demand value lies in the range of 2,250 to 2,750 units per month. A more specific value can be obtained by changing the Demand Start point value in cell E1 and the Demand Increment value in cell E2. For example, if the value of cell E1 is set to 2,250 and the increment in cell E2 is set to 10, the optimal demand value is shown to be between 2,420 and 2,430 units per month.

The breakeven points lie within the ranges 750–1,000 units per month and 3,750–4,000 units per month, as indicated by the change in sign of profit. Again, by changing the values in cells E1 and E2, we can obtain more exact values of the breakeven points.

Figure 2-5(b) is a graphical display of the Total Revenue, Total Expense, and Profit functions for the range of demand values given in column A of Figure 2-5(a). This graph enables us to see how profit changes as demand increases. The optimal demand value (maximum point of the profit curve) appears to be around 2,500 units per month.

Figure 2-5(b) is also a graphical representation of the breakeven points. By graphing the total revenue and total cost curves separately, we can easily identify the breakeven points (the intersection of these two functions). From the graph, the range of profitable demand is approximately 1,000 to 4,000 units per month. Notice also that, at these demand values, the profit curve crosses the x-axis ($0$).
### Table of profit values for a range of demand values

<table>
<thead>
<tr>
<th>Demand</th>
<th>Price per Unit</th>
<th>Total Revenue</th>
<th>Total Expense</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$180</td>
<td>$73,000</td>
<td></td>
<td>$73,000</td>
</tr>
<tr>
<td>250</td>
<td>$175</td>
<td>$43,750</td>
<td>$93,750</td>
<td>$60,000</td>
</tr>
<tr>
<td>500</td>
<td>$170</td>
<td>$66,000</td>
<td>$114,000</td>
<td>$49,000</td>
</tr>
<tr>
<td>750</td>
<td>$165</td>
<td>$123,750</td>
<td>$135,250</td>
<td>$11,500</td>
</tr>
<tr>
<td>1000</td>
<td>$160</td>
<td>$160,000</td>
<td>$156,000</td>
<td>$4,000</td>
</tr>
<tr>
<td>1250</td>
<td>$155</td>
<td>$193,750</td>
<td>$176,750</td>
<td>$17,000</td>
</tr>
<tr>
<td>1500</td>
<td>$150</td>
<td>$225,000</td>
<td>$197,500</td>
<td>$27,500</td>
</tr>
<tr>
<td>1750</td>
<td>$145</td>
<td>$253,750</td>
<td>$218,250</td>
<td>$35,500</td>
</tr>
<tr>
<td>2000</td>
<td>$140</td>
<td>$280,000</td>
<td>$239,000</td>
<td>$41,000</td>
</tr>
<tr>
<td>2250</td>
<td>$135</td>
<td>$303,750</td>
<td>$259,750</td>
<td>$44,000</td>
</tr>
<tr>
<td>2500</td>
<td>$130</td>
<td>$325,000</td>
<td>$280,500</td>
<td>$44,500</td>
</tr>
<tr>
<td>2750</td>
<td>$125</td>
<td>$343,750</td>
<td>$301,250</td>
<td>$42,500</td>
</tr>
<tr>
<td>3000</td>
<td>$120</td>
<td>$360,000</td>
<td>$322,000</td>
<td>$38,000</td>
</tr>
<tr>
<td>3250</td>
<td>$115</td>
<td>$373,750</td>
<td>$342,750</td>
<td>$31,000</td>
</tr>
<tr>
<td>3500</td>
<td>$110</td>
<td>$385,000</td>
<td>$363,500</td>
<td>$21,500</td>
</tr>
<tr>
<td>3750</td>
<td>$105</td>
<td>$393,750</td>
<td>$394,250</td>
<td>$9,000</td>
</tr>
<tr>
<td>4000</td>
<td>$100</td>
<td>$400,000</td>
<td>$405,000</td>
<td>$5,000</td>
</tr>
<tr>
<td>4250</td>
<td>$95</td>
<td>$413,750</td>
<td>$425,750</td>
<td>$22,000</td>
</tr>
<tr>
<td>4500</td>
<td>$90</td>
<td>$420,000</td>
<td>$440,500</td>
<td>$41,500</td>
</tr>
<tr>
<td>4750</td>
<td>$85</td>
<td>$433,750</td>
<td>$462,250</td>
<td>$18,500</td>
</tr>
<tr>
<td>5000</td>
<td>$80</td>
<td>$443,750</td>
<td>$480,000</td>
<td>$80,000</td>
</tr>
<tr>
<td>5250</td>
<td>$75</td>
<td>$393,750</td>
<td>$508,750</td>
<td>$115,050</td>
</tr>
<tr>
<td>5500</td>
<td>$70</td>
<td>$365,000</td>
<td>$529,600</td>
<td>$144,500</td>
</tr>
</tbody>
</table>

**Comment**

As seen in the hand solution to this problem, Equations (2-10) and (2-12) can be used directly to solve for the optimal demand value and breakeven points.
The power of the spreadsheet in this example is the ease with which graphical displays can be generated to support your analysis. Remember, a picture really can be worth a thousand words. Spreadsheets also facilitate sensitivity analysis (to be discussed more fully in Chapter 11). For example, what is the impact on the optimal demand value and breakeven points if variable costs are reduced by 10% per unit? (The new optimal demand value is increased to 2,632 units per month, and the range of profitable demand is widened to 822 to 4,443 units per month.)

**Scenario 2** When the price per unit \( (p) \) for a product or service can be represented more simply as being independent of demand [versus being a linear function of demand, as assumed in Equation (2-1)] and is greater than the variable cost per unit \( (c_v) \), a single breakeven point results. Then, under the assumption that demand is immediately met, total revenue \( (TR) = p \cdot D \). If the linear relationship for costs in Equations (2-7) and (2-8) is also used in the model, the typical situation is depicted in Figure 2-6. This scenario is typified by the Airbus example presented at the beginning of the chapter.
EXAMPLE 2-5  **Breakeven Point When Price Is Independent of Demand**

An engineering consulting firm measures its output in a standard service hour unit, which is a function of the personnel grade levels in the professional staff. The variable cost \( c_v \) is $62 per standard service hour. The charge-out rate [i.e., selling price \( p \)] is $85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost \( CF \) is $2,024,000 per year. For this firm,

(a) what is the breakeven point in standard service hours and in percentage of total capacity?

(b) what is the percentage reduction in the breakeven point (sensitivity) if fixed costs are reduced 10%; if variable cost per hour is reduced 10%; and if the selling price per unit is increased by 10%?

**Solution**

(a)

Total revenue = total cost  \( \text{(breakeven point)} \)

\[
pD' = CF + c_vD'
\]

\[
D' = \frac{CF}{(p - c_v)}, \tag{2-13}
\]

and

\[
D' = \frac{2,024,000}{(85.56 - 62)} = 85,908 \text{ hours per year}
\]

\[
D' = \frac{85,908}{160,000} = 0.537,
\]

or 53.7% of capacity.
(b) A 10\% reduction in $C_F$ gives

$$D' = \frac{0.9(2,024,000)}{(85.56 - 0.9(62))} = 77,318 \text{ hours per year}$$

and

$$\frac{85,908 - 77,318}{85,908} = 0.10,$$

or a 10\% reduction in $D'$. A 10\% reduction in $c_v$ gives

$$D' = \frac{2,024,000}{[85.56 - 0.9(62)]} = 68,011 \text{ hours per year}$$

and

$$\frac{85,908 - 68,011}{85,908} = 0.208,$$

or a 20.8\% reduction in $D'$. A 10\% increase in $p$ gives

$$D' = \frac{2,024,000}{[1.1(85.56) - 62]} = 63,021 \text{ hours per year}$$

and

$$\frac{85,908 - 63,021}{85,908} = 0.266,$$

or a 26.6\% reduction in $D'$.

Thus, the breakeven point is more sensitive to a reduction in variable cost per hour than to the same percentage reduction in the fixed cost. Furthermore, notice that the breakeven point in this example is highly sensitive to the selling price per unit, $p$.

Market competition often creates pressure to lower the breakeven point of an operation; the lower the breakeven point, the less likely that a loss will occur during market fluctuations. Also, if the selling price remains constant (or increases), a larger profit will be achieved at any level of operation above the reduced breakeven point.

### 2.3 Cost-Driven Design Optimization

As discussed in Section 2.1.6, engineers must maintain a life-cycle (i.e., “cradle to grave”) viewpoint as they design products, processes, and services. Such a complete perspective ensures that engineers consider initial investment costs,
CHAPTER 2 / COST CONCEPTS AND DESIGN ECONOMICS

operation and maintenance expenses and other annual expenses in later years, and environmental and social consequences over the life of their designs. In fact, a movement called Design for the Environment (DFE), or “green engineering,” has prevention of waste, improved materials selection, and reuse and recycling of resources among its goals. Designing for energy conservation, for example, is a subset of green engineering. Another example is the design of an automobile bumper that can be easily recycled. As you can see, engineering design is an economically driven art.

Examples of cost minimization through effective design are plentiful in the practice of engineering. Consider the design of a heat exchanger in which tube material and configuration affect cost and dissipation of heat. The problems in this section designated as “cost-driven design optimization” are simple design models intended to illustrate the importance of cost in the design process. These problems show the procedure for determining an optimal design, using cost concepts. We will consider discrete and continuous optimization problems that involve a single design variable, \( X \). This variable is also called a primary cost driver, and knowledge of its behavior may allow a designer to account for a large portion of total cost behavior.

For cost-driven design optimization problems, the two main tasks are as follows:

1. Determine the optimal value for a certain alternative’s design variable. For example, what velocity of an aircraft minimizes the total annual costs of owning and operating the aircraft?
2. Select the best alternative, each with its own unique value for the design variable. For example, what insulation thickness is best for a home in Virginia: R11, R19, R30, or R38?

In general, the cost models developed in these problems consist of three types of costs:

1. fixed cost(s)
2. cost(s) that vary directly with the design variable
3. cost(s) that vary indirectly with the design variable

A simplified format of a cost model with one design variable is

\[
\text{Cost} = aX + \frac{b}{X} + k, \quad (2-14)
\]

where \( a \) is a parameter that represents the directly varying cost(s),
\( b \) is a parameter that represents the indirectly varying cost(s),
\( k \) is a parameter that represents the fixed cost(s), and
\( X \) represents the design variable in question (e.g., weight or velocity).
In a particular problem, the parameters \( a, b, \) and \( k \) may actually represent the sum of a group of costs in that category, and the design variable may be raised to some power for either directly or indirectly varying costs.∗

The following steps outline a general approach for optimizing a design with respect to cost:

1. Identify the design variable that is the primary cost driver (e.g., pipe diameter or insulation thickness).
2. Write an expression for the cost model in terms of the design variable.
3. Set the first derivative of the cost model with respect to the continuous design variable equal to zero. For discrete design variables, compute the value of the cost model for each discrete value over a selected range of potential values.
4. Solve the equation found in Step 3 for the optimum value of the continuous design variable.† For discrete design variables, the optimum value has the minimum cost value found in Step 3. This method is analogous to taking the first derivative for a continuous design variable and setting it equal to zero to determine an optimal value.
5. For continuous design variables, use the second derivative of the cost model with respect to the design variable to determine whether the optimum value found in Step 4 corresponds to a global maximum or minimum.

∗ A more general model is the following: Cost = \( k + aX + b_1Xe_{1} + b_2Xe_{2} + \cdots \), where \( e_1 = -1 \) reflects costs that vary inversely with \( X \), \( e_2 = 2 \) indicates costs that vary as the square of \( X \), and so forth.

† If multiple optima (stationary points) are found in Step 4, finding the global optimum value of the design variable will require a little more effort. One approach is to systematically use each root in the second derivative equation and assign each point as a maximum or a minimum based on the sign of the second derivative. A second approach would be to use each root in the objective function and see which point best satisfies the cost function.

### Example 2-6

**How Fast Should the Airplane Fly?**

The cost of operating a jet-powered commercial (passenger-carrying) airplane varies as the three-halves \((3/2)\) power of its velocity; specifically, \( C_O = knv^{3/2} \), where \( n \) is the trip length in miles, \( k \) is a constant of proportionality, and \( v \) is velocity in miles per hour. It is known that at 400 miles per hour the average cost of operation is $300 per mile. The company that owns the aircraft wants to minimize the cost of operation, but that cost must be balanced against the cost of passengers’ time \((C_C)\), which has been set at $300,000 per hour.

(a) At what velocity should the trip be planned to minimize the total cost, which is the sum of the cost of operating the airplane and the cost of passengers’ time?

(b) How do you know that your answer for the problem in Part (a) minimizes the total cost?
Solution

(a) The equation for total cost ($CT$) is

$$CT = CO + CC = kn^{3/2} + ($300,000\text{ per hour}) \left(\frac{n}{v}\right),$$

where $n/v$ has time (hours) as its unit.

Now we solve for the value of $k$:

$$\frac{CO}{n} = kv^{3/2}$$

$$\frac{$300}{\text{mile}} = k \left(\frac{400\text{ miles}}{\text{hour}}\right)^{3/2}$$

$$k = \frac{$300/\text{mile}}{\left(\frac{400\text{ miles}}{\text{hour}}\right)^{3/2}}$$

$$k = \frac{$300/\text{mile}}{8000\left(\frac{\text{miles}^{3/2}}{\text{hour}^{1/2}}\right)}$$

$$k = $0.0375\text{ hours}^{3/2}\text{ miles}^{5/2}.$$

Thus,

$$CT = \left($0.0375\text{ hours}^{3/2}\text{ miles}^{5/2}\right) (n \text{ miles}) \left(\frac{v \text{ miles}}{\text{hour}}\right)^{3/2} + (300,000 \text{ per hour}) \left(\frac{n \text{ miles}}{v \text{ miles} \text{hour}}\right)$$

$$CT = $0.0375nv^{3/2} + $300,000 \left(\frac{n}{v}\right).$$

Next, the first derivative is taken:

$$\frac{dCT}{dv} = \frac{3}{2} \left($0.0375\text{ hours}^{1/2}\text{ miles}^{5/2}\right) - \frac{$300,000n}{v^2} = 0.$$

So,

$$0.05625v^{1/2} - \frac{300,000}{v^2} = 0$$

$$0.05625v^{5/2} - 300,000 = 0$$

$$v^{5/2} = \frac{300,000}{0.05625} = 5,333,333$$

$$v^* = (5,333,333)^{0.4} = 490.68 \text{ mph}.$$
(b) Finally, we check the second derivative to confirm a minimum cost solution:

\[
\frac{d^2 C_T}{dv^2} = \frac{0.028125}{v^{1/2}} + \frac{600,000}{v^3}
\]

for \( v > 0 \), and therefore, \( \frac{d^2 C_T}{dv^2} > 0 \).

The company concludes that \( v = 490.68 \) mph minimizes the total cost of this particular airplane’s flight.

---

**EXAMPLE 2-7 Energy Savings through Increased Insulation**

This example deals with a discrete optimization problem of determining the most economical amount of attic insulation for a large single-story home in Virginia. In general, the heat lost through the roof of a single-story home is

\[
\text{Heat loss in Btu per hour} = \left( \frac{\Delta \text{Temperature}}{\text{in } ^\circ \text{F}} \right) \left( \frac{\text{Area in ft}^2}{\text{in ft}^2} \right) \left( \frac{\text{Conductance in Btu/hour ft}^2 - ^\circ \text{F}}{\text{Btu/hour ft}^2 - ^\circ \text{F}} \right),
\]

or

\[
Q = (T_{\text{in}} - T_{\text{out}}) \cdot A \cdot U.
\]

In southwest Virginia, the number of heating days per year is approximately 230, and the annual heating degree-days equals 230 \((65^\circ \text{F} - 46^\circ \text{F}) = 4,370 \) degree-days per year. Here \( 65^\circ \text{F} \) is assumed to be the average inside temperature and \( 46^\circ \text{F} \) is the average outside temperature each day.

Consider a 2,400-ft\(^2\) single-story house in Blacksburg. The typical annual space-heating load for this size of a house is \( 100 \times 10^6 \) Btu. That is, with no insulation in the attic, we lose about \( 100 \times 10^6 \) Btu per year.* Common sense dictates that the “no insulation” alternative is not attractive and is to be avoided.

With insulation in the attic, the amount of heat lost each year will be reduced. The value of energy savings that results from adding insulation and reducing heat loss is dependent on what type of residential heating furnace is installed. For this example, we assume that an electrical resistance furnace is installed by the builder, and its efficiency is near 100%.

Now we’re in a position to answer the following question: What amount of insulation is most economical? An additional piece of data we need involves the cost of electricity, which is $0.074 per kWh. This can be converted to dollars per \( 10^6 \) Btu as follows (1 kWh = 3,413 Btu):

\[
\frac{kWh}{3,413 \text{ Btu}} = 293 \text{ kWh per million Btu}
\]

* \( 100 \times 10^6 \) Btu/yr \( \approx \) \( \frac{4,370 \ \text{F-days per year}}{1.00 \ \text{efficiency}} \) \( \times \) \( \frac{0.397 \text{ Btu/hr ft}^2 - \text{F}}{24 \text{ hours/day}} \), where 0.397 is the U-factor with no insulation.
293 kWh \frac{10^6 Btu}{kWh} \approx 21.75/10^6 Btu.

The cost of several insulation alternatives and associated space-heating loads for this house are given in the following table:

<table>
<thead>
<tr>
<th>Amount of Insulation</th>
<th>R11</th>
<th>R19</th>
<th>R30</th>
<th>R38</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment cost ($)</td>
<td>600</td>
<td>900</td>
<td>1,300</td>
<td>1,600</td>
</tr>
<tr>
<td>Annual heating load (Btu/year)</td>
<td>$74 \times 10^6$</td>
<td>$69.8 \times 10^6$</td>
<td>$67.2 \times 10^6$</td>
<td>$66.2 \times 10^6$</td>
</tr>
</tbody>
</table>

In view of these data, which amount of attic insulation is most economical? The life of the insulation is estimated to be 25 years.

**Solution**

Set up a table to examine total life-cycle costs:

<table>
<thead>
<tr>
<th></th>
<th>R11</th>
<th>R19</th>
<th>R30</th>
<th>R38</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>Investment cost</td>
<td>$600</td>
<td>$900</td>
<td>$1,300</td>
</tr>
<tr>
<td>B.</td>
<td>Cost of heat loss per year</td>
<td>$1,609.50</td>
<td>$1,518.15</td>
<td>$1,461.60</td>
</tr>
<tr>
<td>C.</td>
<td>Cost of heat loss over 25 years</td>
<td>$40,237.50</td>
<td>$37,953.75</td>
<td>$36,540</td>
</tr>
<tr>
<td>D.</td>
<td>Total life cycle costs (A + C)</td>
<td>$40,837.50</td>
<td>$38,853.75</td>
<td>$37,840</td>
</tr>
</tbody>
</table>

Answer: To minimize total life-cycle costs, select R38 insulation.

**Caution**

This conclusion may change when we consider the time value of money (i.e., an interest rate greater than zero) in Chapter 4. In such a case, it will not necessarily be true that adding more and more insulation is the optimal course of action.

### 2.4 Present Economy Studies

When alternatives for accomplishing a specific task are being compared over *one year or less* and the influence of time on money can be ignored, engineering economic analyses are referred to as *present economy studies*. Several situations involving present economy studies are illustrated in this section. The rules, or criteria, shown next will be used to select the preferred alternative when defect-free output (yield) is *variable* or constant among the alternatives being considered.
RULE 1: When revenues and other economic benefits are present and vary among alternatives, choose the alternative that maximizes overall profitability based on the number of defect-free units of a product or service produced.

RULE 2: When revenues and other economic benefits are not present or are constant among all alternatives, consider only the costs and select the alternative that minimizes total cost per defect-free unit of product or service output.

2.4.1 Total Cost in Material Selection

In many cases, economic selection among materials cannot be based solely on the costs of materials. Frequently, a change in materials will affect the design and processing costs, and shipping costs may also be altered.

EXAMPLE 2-8 Choosing the Most Economic Material for a Part

A good example of this situation is illustrated by a part for which annual demand is 100,000 units. The part is produced on a high-speed turret lathe, using 1112 screw-machine steel costing $0.30 per pound. A study was conducted to determine whether it might be cheaper to use brass screw stock, costing $1.40 per pound. Because the weight of steel required per piece was 0.0353 pounds and that of brass was 0.0384 pounds, the material cost per piece was $0.0106 for steel and $0.0538 for brass. However, when the manufacturing engineering department was consulted, it was found that, although 57.1 defect-free parts per hour were being produced by using steel, the output would be 102.9 defect-free parts per hour if brass were used. Which material should be used for this part?

Solution

The machine attendant was paid $15.00 per hour, and the variable (i.e., traceable) overhead costs for the turret lathe were estimated to be $10.00 per hour. Thus, the total-cost comparison for the two materials was as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>1112 Steel</th>
<th>Brass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>$0.30 \times 0.0353 = $0.0106</td>
<td>$1.40 \times 0.0384 = $0.0538</td>
</tr>
<tr>
<td>Labor</td>
<td>$15.00/57.1 = 0.2627</td>
<td>$15.00/102.9 = 0.1458</td>
</tr>
<tr>
<td>Variable overhead</td>
<td>$10.00/57.1 = 0.1751</td>
<td>$10.00/102.9 = 0.0972</td>
</tr>
<tr>
<td>Total cost per piece</td>
<td>$0.4484</td>
<td>$0.2968</td>
</tr>
<tr>
<td>Saving per piece by use of brass</td>
<td>$0.4484 - $0.2968 = $0.1516</td>
<td></td>
</tr>
</tbody>
</table>
Because 100,000 parts are made each year, revenues are constant across the alternatives. Rule 2 would select brass, and its use will produce a savings of $151.60 per thousand (a total of $15,160 for the year). It is also clear that costs other than the cost of material were important in the study.

Care should be taken in making economic selections between materials to ensure that any differences in shipping costs, yields, or resulting scrap are taken into account. Commonly, alternative materials do not come in the same stock sizes, such as sheet sizes and bar lengths. This may considerably affect the yield obtained from a given weight of material. Similarly, the resulting scrap may differ for various materials.

In addition to deciding what material a product should be made of, there are often alternative methods or machines that can be used to produce the product, which, in turn, can impact processing costs. Processing times may vary with the machine selected, as may the product yield. As illustrated in Example 2-9, these considerations can have important economic implications.

### EXAMPLE 2-9 Choosing the Most Economical Machine for Production

Two currently owned machines are being considered for the production of a part. The capital investment associated with the machines is about the same and can be ignored for purposes of this example. The important differences between the machines are their production capacities (production rate $\times$ available production hours) and their reject rates (percentage of parts produced that cannot be sold). Consider the following table:

<table>
<thead>
<tr>
<th></th>
<th>Machine A</th>
<th>Machine B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production rate</td>
<td>100 parts/hour</td>
<td>130 parts/hour</td>
</tr>
<tr>
<td>Hours available for production</td>
<td>7 hours/day</td>
<td>6 hours/day</td>
</tr>
<tr>
<td>Percent parts rejected</td>
<td>3%</td>
<td>10%</td>
</tr>
</tbody>
</table>

The material cost is $6.00 per part, and all defect-free parts produced can be sold for $12 each. (Rejected parts have negligible scrap value.) For either machine, the operator cost is $15.00 per hour and the variable overhead rate for traceable costs is $5.00 per hour.

(a) Assume that the daily demand for this part is large enough that all defect-free parts can be sold. Which machine should be selected?

(b) What would the percent of parts rejected have to be for Machine B to be as profitable as Machine A?
Solution

(a) Rule 1 applies in this situation because total daily revenues (selling price per part times the number of parts sold per day) and total daily costs will vary depending on the machine chosen. Therefore, we should select the machine that will maximize the profit per day:

\[
\text{Profit per day} = \text{Revenue per day} - \text{Cost per day} = (\text{Production rate})(\text{Production hours})(\$12/\text{part}) \\
\times [1 - (\%\text{rejected}/100)] - (\text{Production rate})(\text{Production hours})(\$6/\text{part}) \\
- (\text{Production hours})(\$15/\text{hour} + \$5/\text{hour}).
\]

Machine A: Profit per day = \[
\left( \frac{100 \text{ parts}}{\text{hour}} \right) \left( \frac{7 \text{ hours}}{\text{day}} \right) \left( \frac{\$12}{\text{part}} \right) (1 - 0.03) \\
- \left( \frac{100 \text{ parts}}{\text{hour}} \right) \left( \frac{7 \text{ hours}}{\text{day}} \right) \left( \frac{\$6}{\text{part}} \right) \\
- \left( \frac{7 \text{ hours}}{\text{day}} \right) \left( \frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}} \right)
\]

= $3,808 per day.

Machine B: Profit per day = \[
\left( \frac{130 \text{ parts}}{\text{hour}} \right) \left( \frac{6 \text{ hours}}{\text{day}} \right) \left( \frac{\$12}{\text{part}} \right) (1 - 0.10) \\
- \left( \frac{130 \text{ parts}}{\text{hour}} \right) \left( \frac{6 \text{ hours}}{\text{day}} \right) \left( \frac{\$6}{\text{part}} \right) \\
- \left( \frac{6 \text{ hours}}{\text{day}} \right) \left( \frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}} \right)
\]

= $3,624 per day.

Therefore, select Machine A to maximize profit per day.

(b) To find the breakeven percent of parts rejected, \(X\), for Machine B, set the profit per day of Machine A equal to the profit per day of Machine B, and solve for \(X\):

\[
\frac{\$3,808}{\text{day}} = \left( \frac{130 \text{ parts}}{\text{hour}} \right) \left( \frac{6 \text{ hours}}{\text{day}} \right) \left( \frac{\$12}{\text{part}} \right) (1 - X) - \left( \frac{130 \text{ parts}}{\text{hour}} \right) \\
\times \left( \frac{6 \text{ hours}}{\text{day}} \right) \left( \frac{\$6}{\text{part}} \right) - \left( \frac{6 \text{ hours}}{\text{day}} \right) \left( \frac{\$15}{\text{hour}} + \frac{\$5}{\text{hour}} \right).
\]

Thus, \(X = 0.08\), so the percent of parts rejected for Machine B can be no higher than 8% for it to be as profitable as Machine A.
2.4.2 Alternative Machine Speeds

Machines can frequently be operated at various speeds, resulting in different rates of product output. However, this usually results in different frequencies of machine downtime to permit servicing or maintaining the machine, such as resharpening or adjusting tooling. Such situations lead to present economy studies to determine the preferred operating speed. We first assume that there is an unlimited amount of work to be done in Example 2-10. Secondly, Example 2-11 illustrates how to deal with a fixed (limited) amount of work.

EXAMPLE 2-10 Best Operating Speed for an Unlimited Amount of Work

A simple example of alternative machine speeds involves the planing of lumber. Lumber put through the planer increases in value by $0.10 per board foot. When the planer is operated at a cutting speed of 5,000 feet per minute, the blades have to be sharpened after 2 hours of operation, and the lumber can be planed at the rate of 1,000 board-feet per hour. When the machine is operated at 6,000 feet per minute, the blades have to be sharpened after 1 1/2 hours of operation, and the rate of planing is 1,200 board-feet per hour. Each time the blades are changed, the machine has to be shut down for 15 minutes. The blades, unsharpened, cost $50 per set and can be sharpened 10 times before having to be discarded. Sharpening costs $10 per occurrence. The crew that operates the planer changes and resets the blades. At what speed should the planer be operated?

Solution

Because the labor cost for the crew would be the same for either speed of operation and because there was no discernible difference in wear upon the planer, these factors did not have to be included in the study.

In problems of this type, the operating time plus the delay time due to the necessity for tool changes constitute a cycle time that determines the output from the machine. The time required for a complete cycle determines the number of cycles that can be accomplished in a period of available time (e.g., one day), and a certain portion of each complete cycle is productive. The actual productive time will be the product of the productive time per cycle and the number of cycles per day.

<table>
<thead>
<tr>
<th>Value per day ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 5,000 feet per minute</td>
</tr>
<tr>
<td>Cycle time = 2 hours + 0.25 hour = 2.25 hours</td>
</tr>
<tr>
<td>Cycles per day = 8 ÷ 2.25 = 3.555</td>
</tr>
<tr>
<td>Value added by planing = 3.555 × 2 × 1,000 × $0.10 = 711.00</td>
</tr>
<tr>
<td>Cost of resharpening blades = 3.555 × $10 = $35.55</td>
</tr>
<tr>
<td>Cost of blades = 3.555 × $50/10 = 17.78</td>
</tr>
<tr>
<td>Total cost cash flow = −53.33</td>
</tr>
<tr>
<td>Net increase in value (profit) per day = 657.67</td>
</tr>
</tbody>
</table>
At 6,000 feet per minute

Cycle time = 1.5 \text{ hours} + 0.25 \text{ hour} = 1.75 \text{ hours}
Cycles per day = 8 \div 1.75 = 4.57
Value added by planing = 4.57 \times 1.5 \times 1,200 \times 0.10 = 822.60^*
Cost of resharpening blades = 4.57 \times 10 = 45.70
Cost of blades = 4.57 \times 50 \div 10 = 22.85
Total cost cash flow = -68.55
Net increase in value (profit) per day = 754.05

* The units work out as follows: (cycles/day)(hours/cycle)(board feet/hour)(dollar value/board-foot) = dollars/day.

Thus, in Example 2-10 it is more economical according to Rule 1 to operate at 6,000 feet per minute, in spite of the more frequent sharpening of blades that is required.

EXAMPLE 2-11 Fixed Amount of Work: Now Which Speed Is Best?

Example 2-10 assumed that every board-foot of lumber that is planed can be sold. If there is limited demand for the lumber, a correct choice of machining speeds can be made with Rule 2 by minimizing total cost per unit of output. Suppose now we want to know the better machining speed when only one job requiring 6,000 board-feet of planing is considered. Solve by using a spreadsheet.

Spreadsheet Solution

For a fixed planing requirement of 6,000 board-feet, the value added by planing is 6,000 ($0.10) = $600 for either cutting speed. Hence, we want to minimize total cost per board-foot planed.

The total cost per board-foot planed is a combination of the blade cost and resharpening cost. These costs are most easily stated on a per cycle basis (blade cost/cycle = $50/10 cycles and resharpening cost/cycle = $10/cycle). Now the total cost for a fixed job length can be determined by the number of cycles required.

Figure 2-7 presents a spreadsheet model for this problem. The cell formulas were developed by using the cycle time solution approach of Example 2-10. The production rate per hour (cells E1 and F1) is converted to a production rate per cycle (cells B10 and C10). This value is used to determine the number of cycles required to complete the fixed length job (cells B11 and C11).

For a 6,000-board-foot job, select the slower cutting speed (5,000 feet per minute) to minimize cost. During the 0.92 hour of time savings for the 6,000-feet-per-minute cutting speed, we assume that the operator is idle.
2.4.3 Making versus Purchasing (Outsourcing) Studies*

In the short run, say, one year or less, a company may consider producing an item in-house even though the item can be purchased (outsourced) from a supplier at a price lower than the company’s standard production costs. (See Section 2.1.2.) This could occur if (1) direct, indirect, and overhead costs are incurred regardless of whether the item is purchased from an outside supplier and (2) the incremental cost of producing an item in the short run is less than the supplier’s price. Therefore, the relevant short-run costs of make versus purchase decisions are the incremental costs incurred and the opportunity costs of the resources involved.

* Much interest has been shown in outsourcing decisions. For example, see P. Chalos, “Costing, Control, and Strategic Analysis in Outsourcing Decisions,” Journal of Cost Management, 8, no. 4 (Winter 1995): 31–37.
Opportunity costs may become significant when in-house manufacture of an item causes other production opportunities to be forgone (often because of insufficient capacity). But in the long run, capital investments in additional manufacturing plant and capacity are often feasible alternatives to outsourcing. (Much of this book is concerned with evaluating the economic worthiness of proposed capital investments.) Because engineering economy often deals with changes to existing operations, standard costs may not be too useful in make-versus-purchase studies. In fact, if they are used, standard costs can lead to uneconomical decisions. Example 2-12 illustrates the correct procedure to follow in performing make-versus-purchase studies based on incremental costs.

### Example 2-12

**To Produce or Not to Produce?—That Is the Question**

A manufacturing plant consists of three departments: A, B, and C. Department A occupies 100 square meters in one corner of the plant. Product X is one of several products being produced in Department A. The daily production of Product X is 576 pieces. The cost accounting records show the following average daily production costs for Product X:

<table>
<thead>
<tr>
<th>Cost Item</th>
<th>Description</th>
<th>Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Labor</td>
<td>(1 operator working 4 hours per day at $22.50/hr, including fringe benefits, plus a part-time foreman at $30 per day)</td>
<td>$120.00</td>
</tr>
<tr>
<td>Direct Material</td>
<td></td>
<td>$86.40</td>
</tr>
<tr>
<td>Overhead</td>
<td>(at $0.82 per square meter of floor area)</td>
<td>$82.00</td>
</tr>
<tr>
<td><strong>Total cost per day</strong></td>
<td></td>
<td><strong>$288.40</strong></td>
</tr>
</tbody>
</table>

The department foreman has recently learned about an outside company that sells Product X at $0.35 per piece. Accordingly, the foreman figured a cost per day of $0.35(576) = $201.60, resulting in a daily savings of $288.40 − $201.60 = $86.80. Therefore, a proposal was submitted to the plant manager for shutting down the production line of Product X and buying it from the outside company.

However, after examining each component separately, the plant manager decided not to accept the foreman’s proposal based on the unit cost of Product X:

1. **Direct Labor:** Because the foreman was supervising the manufacture of other products in Department A in addition to Product X, the only possible savings in labor would occur if the operator working 4 hours per day on Product X were not reassigned after this line is shut down. That is, a maximum savings of $90.00 per day would result.

2. **Materials:** The maximum savings on direct material will be $86.40. However, this figure could be lower if some of the material for Product X is obtained from scrap of another product.
3. **Overhead:** Because other products are made in Department A, no reduction in total floor space requirements will probably occur. Therefore, no reduction in overhead costs will result from discontinuing Product X. It has been estimated that there will be daily savings in the variable overhead costs traceable to Product X of about $3.00 due to a reduction in power costs and in insurance premiums.

**Solution**

If the manufacture of Product X is discontinued, the firm will save at most $90.00 in direct labor, $86.40 in direct materials, and $3.00 in variable overhead costs, which totals $179.40 per day. This estimate of actual cost savings per day is less than the potential savings indicated by the cost accounting records ($288.40 per day), and it would not exceed the $201.60 to be paid to the outside company if Product X is purchased. For this reason, the plant manager used Rule 2 and rejected the proposal of the foreman and continued the manufacture of Product X.

In conclusion, Example 2-12 shows how an erroneous decision might be made by using the unit cost of Product X from the cost accounting records without detailed analysis. The fixed cost portion of Product X’s unit cost, which is present even if the manufacture of Product X is discontinued, was not properly accounted for in the original analysis by the foreman.

### 2.4.4 Trade-Offs in Energy Efficiency Studies

Energy efficiency affects the annual expense of operating an electrical device such as a pump or motor. Typically, a more energy-efficient device requires a higher capital investment than does a less energy-efficient device, but the extra capital investment usually produces annual savings in electrical power expenses relative to a second pump or motor that is less energy efficient. This important trade-off between capital investment and annual electric power consumption will be considered in several chapters of this book. Hence, the purpose of Section 2.4.4 is to explain how the annual expense of operating an electrical device is calculated and traded off against capital investment cost.

If an electric pump, for example, can deliver a given horsepower (hp) or kiloWatt (kW) rating to an industrial application, the _input_ energy requirement is determined by dividing the given output by the energy efficiency of the device. The input requirement in hp or kW is then multiplied by the annual hours that the device operates and the unit cost of electric power. You can see that the higher the efficiency of the pump, the lower the annual cost of operating the device is relative to another less-efficient pump.
**EXAMPLE 2-13 Investing In Electrical Efficiency**

Two pumps capable of delivering 100 hp to an agricultural application are being evaluated in a present economy study. The selected pump will only be utilized for one year, and it will have no market value at the end of the year. Pertinent data are summarized as follows:

<table>
<thead>
<tr>
<th></th>
<th>ABC Pump</th>
<th>XYZ Pump</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase price</td>
<td>$2,900</td>
<td>$6,200</td>
</tr>
<tr>
<td>Annual maintenance</td>
<td>$170</td>
<td>$510</td>
</tr>
<tr>
<td>Efficiency</td>
<td>80%</td>
<td>90%</td>
</tr>
</tbody>
</table>

If electric power costs $0.10 per kWh and the pump will be operated 4,000 hours per year, which pump should be chosen? Recall that 1 hp = 0.746 kW.

**Solution**

The annual expense of electric power for the ABC pump is

\[
(100 \text{ hp}/0.80)(0.746 \text{ kW/hp})(0.10 \text{ /kWh})(4,000 \text{ hours/yr}) = 37,300.
\]

For the XYZ Pump, the annual expense of electric power is

\[
(100 \text{ hp}/0.90)(0.746 \text{ kW/hp})(0.10 \text{ /kWh})(4,000 \text{ hours/yr}) = 33,156.
\]

Thus, the total annual cost of owning and operating the ABC pump is $40,370, while the total cost of owning and operating the XYZ pump for one year is $39,866. Consequently, the more energy-efficient XYZ pump should be selected to minimize total annual cost. Notice the difference in annual energy expense ($4,144) that results from a 90% efficient pump relative to an 80% efficient pump. This cost reduction more than balances the extra $3,300 in capital investment and $340 in annual maintenance required for the XYZ pump.

**2.5 CASE STUDY—The Economics of Daytime Running Lights**

The use of Daytime Running Lights (DRLs) has increased in popularity with car designers throughout the world. In some countries, motorists are required to drive with their headlights on at all times. U.S. car manufacturers now offer models equipped with daytime running lights. Most people would agree that driving with the headlights on at night is cost effective with respect to extra fuel consumption and safety considerations (not to mention required by law!). Cost effective means that benefits outweigh (exceed) the costs. However, some consumers have questioned whether it is cost effective to drive with your headlights on during the day.

In an attempt to provide an answer to this question, let us consider the following suggested data:

- 75% of driving takes place during the daytime.
- 2% of fuel consumption is due to accessories (radio, headlights, etc.).
Cost of fuel = $3.00 per gallon.
Average distance traveled per year = 15,000 miles.
Average cost of an accident = $2,800.
Purchase price of headlights = $25.00 per set (2 headlights).
Average time car is in operation per year = 350 hours.
Average life of a headlight = 200 operating hours.
Average fuel consumption = 1 gallon per 30 miles.

Let’s analyze the cost-effectiveness of driving with headlights on during the day by considering the following set of questions:

• What are the extra costs associated with driving with headlights on during the day?
• What are the benefits associated with driving with headlights on during the day?
• What additional assumptions (if any) are needed to complete the analysis?
• Is it cost effective to drive with headlights on during the day?

Solution

After some reflection on the above questions, you could reasonably contend that the extra costs of driving with headlights on during the day include increased fuel consumption and more frequent headlight replacement. Headlights increase visibility to other drivers on the road. Another possible benefit is the reduced chance of an accident.

Additional assumptions needed to consider during our analysis of the situation include:

1. the percentage of fuel consumption due to headlights alone
2. how many accidents can be avoided per unit time.

Selecting the dollar as our common unit of measure, we can compute the extra cost associated with daytime use of headlights and compare it to the expected benefit (also measured in dollars). As previously determined, the extra costs include increased fuel consumption and more frequent headlight replacement. Let’s develop an estimate of the annual fuel cost:

\[
\text{Annual fuel cost} = (15,000 \text{ mi/yr})(1 \text{ gal/30 mi})(3.00 \text{ /gal}) = 1,500 \text{ /yr.}
\]

Assume (worst case) that 2% of fuel consumption is due to normal (night-time) use of headlights.

Fuel cost due to normal use of headlights = $(1,500 \text{ /yr})(0.02) = 30 \text{ /yr.}$

Fuel cost due to continuous use of headlights = $(4)(30 \text{ /yr}) = 120 \text{ /yr.}$

Headlight cost for normal use = $(0.25) \left( \frac{350 \text{ hours/yr}}{200 \text{ hours/set}} \right) \left( \frac{25 \text{ /set}}{\text{set}} \right) = 10.94 \text{ /yr.}$
Headlight cost for continuous use = \( \left( \frac{350 \text{ hours/yr}}{200 \text{ hours/set}} \right) \left( \frac{\$25}{\text{set}} \right) = \$43.75/\text{yr.} \)

Total cost associated with daytime use = \( \left( \$120 - \$30 \right) + \left( \$43.75 - \$10.94 \right) = \$122.81/\text{yr.} \)

If driving with your headlights on during the day results in at least one accident being avoided during the next \( \frac{\$2,800}{\$122.81} = 22.8 \) years, then the continuous use of your headlights is cost effective. Although in the short term, you may be able to contend that the use of DRLs lead to increased fuel and replacement bulb costs, the benefits of increased personal safety and mitigation of possible accident costs in the long-run more than offset the apparent short-term cost savings.

2.6 Summary

In this chapter, we have discussed cost terminology and concepts important in engineering economy. It is important that the meaning and use of various cost terms and concepts be understood in order to communicate effectively with other engineering and management personnel. A listing of important abbreviations and notation, by chapter, is provided in Appendix B.

Several general economic concepts were discussed and illustrated. First, the ideas of consumer and producer goods and services, measures of economic growth, competition, and necessities and luxuries were covered. Then, some relationships among costs, price, and volume (demand) were discussed. Included were the concepts of optimal volume and breakeven points. Important economic principles of design optimization were also illustrated in this chapter.

The use of present-economy studies in engineering decision making can provide satisfactory results and save considerable analysis effort. When an adequate engineering economic analysis can be accomplished by considering the various monetary consequences that occur in a short time period (usually one year or less), a present-economy study should be used.

Problems

The number(s) in color at the end of a problem refer to the section(s) in that chapter most closely related to the problem.

2-1. A company in the process industry produces a chemical compound that is sold to manufacturers for use in the production of certain plastic products. The plant that produces the compound employs approximately 300 people. Develop a list of six different cost elements that would be fixed and a similar list of six cost elements that would be variable. (2.1)

2-2. Classify each of the following cost items as mostly fixed or variable: (2.1)

<table>
<thead>
<tr>
<th>Fixed</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw materials</td>
<td>Administrative salaries</td>
</tr>
<tr>
<td>Direct labor</td>
<td>Payroll taxes</td>
</tr>
<tr>
<td>Depreciation</td>
<td>Insurance (building and equipment)</td>
</tr>
<tr>
<td>Supplies</td>
<td>Clerical salaries</td>
</tr>
<tr>
<td>Utilities</td>
<td>Sales commissions</td>
</tr>
<tr>
<td>Property taxes</td>
<td>Rent</td>
</tr>
<tr>
<td>Interest on borrowed money</td>
<td></td>
</tr>
</tbody>
</table>
2-3. A group of enterprising engineering students has developed a process for extracting combustible methane gas from cow manure (don’t worry, the exhaust is orderless). With a specially adapted internal combustion engine, the students claim that an automobile can be propelled 15 miles per day from the “cow gas” produced by a single cow. Their experimental car can travel 60 miles per day for an estimated cost of $5 (this is the allocated cost of the methane process equipment—the cow manure is essentially free). (2.1)

a. How many cows would it take to fuel 1,000,000 miles of annual driving by a fleet of cars? What is the annual cost?

b. How does your answer to Part (a) compare to a gasoline-fueled car averaging 30 miles per gallon when the cost of gasoline is $3.00 per gallon?

2-4. A municipal solid-waste site for a city must be located at Site A or Site B. After sorting, some of the solid refuse will be transported to an electric power plant where it will be used as fuel. Data for the hauling of refuse from each site to the power plant are shown in Table P2-4.

<table>
<thead>
<tr>
<th>TABLE P2-4 Table for Problem 2-4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Site A</td>
</tr>
<tr>
<td>Average hauling distance</td>
</tr>
<tr>
<td>Annual rental fee for solid-waste site</td>
</tr>
<tr>
<td>Hauling cost</td>
</tr>
<tr>
<td>Site B</td>
</tr>
<tr>
<td>Average hauling distance</td>
</tr>
<tr>
<td>Annual rental fee for solid-waste site</td>
</tr>
<tr>
<td>Hauling cost</td>
</tr>
</tbody>
</table>

If the power plant will pay $8.00 per cubic yard of sorted solid waste delivered to the plant, where should the solid-waste site be located? Use the city’s viewpoint and assume that 200,000 cubic yards of refuse will be hauled to the plant for one year only. One site must be selected. (2.1)

2-5. Stan Moneymaker presently owns a 10-year-old automobile with low mileage (78,000 miles). The NADA “blue book” value of the car is $2,500. Unfortunately, the car’s transmission just failed, and Stan decided to spend $1,500 to have it repaired. Now, six months later, Stan has decided to sell the car, and he reasons that his asking price should be $2,500 + $1,500 = $4,000. Comment on the wisdom of Stan’s logic. If he receives an offer for $3,000, should he accept it? Explain your reasoning. (2.1)

2-6. You have been invited by friends to fly to Germany for Octoberfest next year. For international travel, you apply for a passport that costs $97 and is valid for 10 years. After you receive your passport, your travel companions decide to cancel the trip because of “insufficient funds.” You decide to also cancel your travel plans because traveling alone is no fun. Is your passport expense a sunk cost or an opportunity cost? Explain your answer. (2.1)

2-7. Suppose your company has just discovered $100,000 worth (this is the original manufacturing cost) of obsolete inventory in an old warehouse. Your boss asks you to evaluate two options: (1) remachine the obsolete parts at a cost of $30,000 and then hopefully resell them for $30,000, or (2) scrap them for $15,000 cash (which is certain) through a secondhand market. What recommendation would you make to your boss? Explain your reasoning. (2.1)

2-8. A friend of yours has been thinking about quitting her regular day job and going into business for herself. She currently makes $60,000 per year as an employee of the Ajax Company, and she anticipates no raise for at least another year. She believes she can make $200,000 as an independent consultant in six-sigma “black belt” training for large corporations. Her start-up expenses are expected to be $120,000 over the next year. If she decides to keep her current job, what is the expected opportunity cost of this decision? Attempt to balance the pros and cons of the option that your friend is turning away from. (2.1)

2-9. Suppose your wealthy aunt has given you a gift of $25,000. You have come up with three options for spending (or investing) the money. First, you’d like (but do not need) a new car to brighten up your home and social life. Second, you can invest the money in a high-tech firm’s common stock. It is expected to increase in value by 20% per year, but this option is fairly risky. Third, you can put the money into a three-year certificate of deposit with a local bank and earn 6% per year. There is little risk in the third option. (2.1)

a. If you decide to purchase the new car, what is the opportunity cost of this choice? Explain your reasoning.
b. If you invest in the high-tech common stock, what is the opportunity cost of this choice? Explain your reasoning.

2-10. In your own words, describe the life-cycle cost concept. Why is the potential for achieving life-cycle cost savings greatest in the acquisition phase of the life cycle? (2.1)

2-11. A large, profitable commercial airline company flies 737-type aircraft, each with a maximum seating capacity of 132 passengers. Company literature states that the economic breakeven point with these aircraft is 62 passengers. (2.2)

a. Draw a conceptual graph to show total revenue and total costs that this company is experiencing.
b. Identify three types of fixed costs that the airline should carefully examine to lower its breakeven point. Explain your reasoning.
c. Identify three types of variable costs that can possibly be reduced to lower the breakeven point. Why did you select these cost items?

2-12. A company produces circuit boards used to update outdated computer equipment. The fixed cost is $42,000 per month, and the variable cost is $53 per circuit board. The selling price per unit is $150 − 0.02D. Maximum output of the plant is 4,000 units per month. (2.2)

a. Determine optimum demand for this product.
b. What is the maximum profit per month?
c. At what volumes does breakeven occur?
d. What is the company’s range of profitable demand?

2-13. A local defense contractor is considering the production of fireworks as a way to reduce dependence on the military. The variable cost per unit is $40. The fixed cost that can be allocated to the production of fireworks is negligible. The price charged per unit will be determined by the equation $p = \$180 - (5)D$, where $D$ represents demand in units sold per week. (2.2)

a. What is the optimum number of units the defense contractor should produce in order to maximize profit per week?
b. What is the profit if the optimum number of units are produced?

2-14. A large wood products company is negotiating a contract to sell plywood overseas. The fixed cost that can be allocated to the production of plywood is $900,000 per month. The variable cost per thousand board feet is $131.50. The price charged will be determined by $p = \$600 - (0.05)D$ per 1,000 board feet. (2.2)

a. For this situation determine the optimal monthly sales volume for this product and calculate the profit (or loss) at the optimal volume.
b. What is domain of profitable demand during a month?

2-15. A company produces and sells a consumer product and is able to control the demand for the product by varying the selling price. The approximate relationship between price and demand is $p = \$38 + \frac{2,700}{D} - \frac{5,000}{D^2}$, for $D > 1$,

where $p$ is the price per unit in dollars and $D$ is the demand per month. The company is seeking to maximize its profit. The fixed cost is $1,000 per month and the variable cost ($cv$) is $40 per unit. (2.2)

a. What is the number of units that should be produced and sold each month to maximize profit?
b. Show that your answer to Part (a) maximizes profit.

2-16. An electric power plant uses solid waste for fuel in the production of electricity. The cost $Y$ in dollars per hour to produce electricity is $Y = 12 + 0.3X + 0.27X^2$, where $X$ is in megawatts. Revenue in dollars per hour from the sale of electricity is $15X - 0.2X^2$. Find the value of $X$ that gives maximum profit. (2.2)

2-17. The annual fixed costs for a plant are $100,000, and the variable costs are $140,000 at 70% utilization of available capacity, with net sales of $280,000. What is the breakeven point in units of production if the selling price per unit is $40? (2.2)

2-18. A plant has a capacity of 4,100 hydraulic pumps per month. The fixed cost is $504,000 per month. The variable cost is $166 per pump, and the sales price is $328 per pump. (Assume that sales equal output volume.) What is the breakeven point in number of pumps per month? What percentage reduction will occur in the breakeven point if fixed costs are reduced by 18% and unit variable costs by 6%? (2.2)

2-19. A cell phone company has a fixed cost of $1,000,000 per month and a variable cost of $20 per month per subscriber. The company charges $29.95 per month to its cell phone customers. (2.2)

a. What is the breakeven point for this company?
b. The company currently has 95,000 subscribers and proposes to raise its monthly fees to $39.95 to cover add-on features such as text messaging, song downloads, game playing, and video watching. What is the new breakeven point if the variable cost increases to $25 per customer per month?

c. If 20,000 subscribers will drop their service because of the monthly fee increase in Part (b), will the company still be profitable?

2-20. A plant operation has fixed costs of $2,000,000 per year, and its output capacity is 100,000 electrical appliances per year. The variable cost is $40 per unit, and the product sells for $90 per unit.

a. Construct the economic breakeven chart.

b. Compare annual profit when the plant is operating at 90% of capacity with the plant operation at 100% capacity. Assume that the first 90% of capacity output is sold at $90 per unit and that the remaining 10% of production is sold at $70 per unit. (2.2)

2-21. A regional airline company estimated four years ago that each pound of aircraft weight adds $30 per year to its fuel expense. Now the cost of jet fuel has doubled from what it was four years ago. A recent engineering graduate employed by the company has made a recommendation to reduce fuel consumption of an aircraft by installing leather seats as part of a “cabin refurbishment program.” The total reduction in weight would be approximately 600 pounds per aircraft. If seats are replaced annually (a worst-case situation), how much can this airline afford to spend on the cabin refurbishments? What nonmonetary advantages might be associated with the refurbishments? Would you support the engineer’s recommendation? (2.1)

2-22. Jerry Smith’s residential air conditioning (AC) system has not been able to keep his house cool enough in 90°F weather. He called his local AC maintenance person, who discovered a leak in the evaporator. The cost to recharge the AC unit is $40 for gas and $45 for labor, but the leak will continue and perhaps grow worse. The AC person cautioned that this service would have to be repeated each year unless the evaporator is replaced. A new evaporator would run about $800–$850.

Jerry reasons that fixing the leak in the evaporator on an annual basis is the way to go. “After all, it will take ten years of leak repairs to equal the evaporator’s replacement cost.” Comment on Jerry’s logic. What would you do? (2.1)

2-23. Ethanol blended with gasoline can be used to power a “flex-fueled” car. One particular blend that is gaining in popularity is E85, which is 85% ethanol and 15% gasoline. E85 is 80% cleaner burning than gasoline alone, and it reduces our dependency on foreign oil. But a flex-fueled car costs $1,000 more than a conventional gasoline-fueled car. Additionally, E85 fuel gets 10% less miles per gallon than a conventional automobile.

Consider a 100% gasoline-fueled car that averages 30 miles per gallon. The E85-fueled car will average about 27 miles per gallon. If either car will be driven 81,000 miles before being traded in, how much will the E85 fuel have to cost (per gallon) to make the flex-fueled car as economically attractive as a conventional gasoline-fueled car? Gasoline costs $2.89 per gallon. Work this problem without considering the time value of money. (2.1)

2-24. The fixed cost for a steam line per meter of pipe is $450 X + $50 per year. The cost for loss of heat from the pipe per meter is $4.8 X^1/2 per year. Here X represents the thickness of insulation in meters, and X is a continuous design variable. (2.3)

a. What is the optimum thickness of the insulation?

b. How do you know that your answer in Part (a) minimizes total cost per year?

c. What is the basic trade-off being made in this problem?

2-25. A farmer estimates that if he harvests his soybean crop now, he will obtain 1,000 bushels, which he can sell at $3.00 per bushel. However, he estimates that this crop will increase by an additional 1,200 bushels of soybeans for each week he delays harvesting, but the price will drop at a rate of 50 cents per bushel per week; in addition, it is likely that he will experience spoilage of approximately 200 bushels per week for each week he delays harvesting. When should he harvest his crop to obtain the largest net cash return, and how much will be received for his crop at that time? (2.3)

2-26. The cost of operating a large ship (C_D) varies as the square of its velocity (v); specifically, C_D = k n v^2, where n is the trip length in miles and k is a constant of proportionality. It is known that at 12 miles/hour the average cost of operation is $100 per mile. The owner of the ship wants to minimize the cost of operation, but it must be balanced against the cost of the perishable cargo (C_c), which the customer has set at $1,500 per hour. At what velocity should the trip be planned to minimize the total cost (C_T), which is the sum of the
cost of operating the ship and the cost of perishable
cargo? (2.3)

2-27. Refer to Example 2-7 on pages 41–42. Which
alternative (insulation thickness) would be most
economical if the cost of insulation triples? Show all
your work. (2.3)

2-28. Suppose you are going on a long trip to your
grandmother’s home in Seattle, 3,000 miles away. You
have decided to drive your old Ford out there, which
gets approximately 18 miles per gallon when cruising
at 70 mph. Because grandma is an excellent cook and
you can stay and eat at her place as long as you want
(for free), you want to get to Seattle as economically as
possible. However, you are also worried about your fuel
consumption rate at high speeds. You also have cost of
food, snacks, and lodging to balance against the cost of
fuel.

What is the optimum average speed you should use
so as to minimize your total trip cost, $C_T$? (2.3)

\[ C_T = C_G + C_{FSS}, \]

where

\[ C_G = n \times p_g \times f \quad (C_G = \text{cost of gas}), \]
\[ C_{FSS} = n \times p_{fss} \times v^{-1} \quad (C_{FSS} = \text{cost of food, snacks, and lodging}), \]

- $n$: trip length (miles),
- $p_g$: gas price, $3.00/gallon,
- $p_{fss}$: average hourly spending money, $5/hour,
- $v$: average Ford velocity (mph),

\[ f = k \times v, \]

where $k$ is a constant of proportionality and $f$ is the fuel
consumption rate in gallons per mile.

2-29. One component of a system’s life-cycle cost is
the cost of system failure. Failure costs can be reduced
by designing a more reliable system. A simplified
expression for system life-cycle cost, $C$, can be written
as a function of the system’s failure rate:

\[ C = \frac{C_I}{\lambda} + C_R \cdot \lambda \cdot t. \]

Here

- $C_I$: investment cost ($ per hour per failure),
- $C_R$: system repair cost,
- $\lambda$: system failure rate
  (failures/operating hour),
- $t$: operating hours.

a. Assume that $C_I$, $C_R$, and $t$ are constants. Derive an
expression for $\lambda$, say $\lambda^*$, that optimizes $C$. (2.3)

b. Does the equation in Part (a) correspond to a
maximum or minimum value of $C$? Show all work
to support your answer.

c. What trade-off is being made in this problem?

2-30. Stan Moneymaker has been shopping for a new
car. He is interested in a certain 4-cylinder sedan that
averages 28 miles per gallon. But the salesperson tried
to persuade Stan that the 6-cylinder model of the same
automobile only costs $2,500 more and is really a “more
sporty and responsive” vehicle. Stan is impressed with
the zip of the 6-cylinder car and reasons that $2,500 is
not too much to pay for the extra power.

How much extra is Stan really paying if the
6-cylinder car averages 22 miles per gallon? Assume
that Stan will drive either automobile 100,000 miles,
gasoline will average $3.00 per gallon, and maintenance
is roughly the same for both cars. State other
assumptions you think are appropriate. (2.4)

2-31. A producer of synthetic motor oil for automobiles
and light trucks has made the following statement:
“One quart of Dynolube added to your next oil change
will increase fuel mileage by one percent. This one-time
additive will improve your fuel mileage over 50,000
miles of driving.” (2.4)

a. Assume the company’s claim is correct. How much
money will be saved by adding one quart of
Dynolube if gasoline costs $3.00 per gallon and
your car averages 20 miles per gallon without the
Dynolube?

b. If a quart of Dynolube sells for $19.95, would you
use this product in your automobile?

2-32. An automobile dealership offers to fill the four
tires of your new car with 100% nitrogen for a cost
of $20. The dealership claims that nitrogen-filled tires
run cooler than those filled with compressed air, and
they advertise that nitrogen extends tire mileage (life)
by 25%. If new tires cost $50 each and are guaranteed
to get 50,000 miles (filled with air) before they require
replacement, is the dealership’s offer a good deal? (2.4)

2-33. In the design of an automobile radiator, an
engineer has a choice of using either a brass–copper
alloy casting or a plastic molding. Either material
provides the same service. However, the brass–copper
alloy casting weighs 25 pounds, compared with 20
pounds for the plastic molding. Every pound of extra
weight in the automobile has been assigned a penalty of
$6 to account for increased fuel consumption during the
life cycle of the car. The brass–copper alloy casting costs
$3.35 per pound, whereas the plastic molding costs $7.40 per pound. Machining costs per casting are $6.00 for the brass–copper alloy. Which material should the engineer select, and what is the difference in unit costs? (2.4)

2-34. Rework Example 2-9 for the case where the capacity of each machine is further reduced by 25% because of machine failures, materials shortages, and operator errors. In this situation, 30,000 units of good (nondefective) product must be manufactured during the next three months. Assume one shift per day and five work days per week. (2.4)

a. Can the order be delivered on time?
b. If only one machine (A or B) can be used in Part (a), which one should it be?

2-35. Two alternative designs are under consideration for a tapered fastening pin. The fastening pins are sold for $0.70 each. Either design will serve equally well and will involve the same material and manufacturing cost except for the lathe and drill operations.

Design A will require 16 hours of lathe time and 4.5 hours of drill time per 1,000 units. Design B will require 7 hours of lathe time and 12 hours of drill time per 1,000 units. The variable operating cost of the lathe, including labor, is $18.60 per hour. The variable operating cost of the drill, including labor, is $16.90 per hour. Finally, there is a sunk cost of $5,000 for Design A and $9,000 for Design B due to obsolete tooling. (2.4)

a. Which design should be adopted if 125,000 units are sold each year?
b. What is the annual saving over the other design?

2-36. A bicycle component manufacturer produces hubs for bike wheels. Two processes are possible for manufacturing, and the parameters of each process are as follows:

<table>
<thead>
<tr>
<th>Process 1</th>
<th>Process 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production rate</td>
<td>35 parts/hour</td>
</tr>
<tr>
<td>Daily production time</td>
<td>4 hours/day</td>
</tr>
<tr>
<td>Percent of parts rejected based on visual inspection</td>
<td>20%</td>
</tr>
</tbody>
</table>

Assume that the daily demand for hubs allows all defect-free hubs to be sold. Additionally, tested or rejected hubs cannot be sold.

Find the process that maximizes profit per day if each part is made from $4 worth of material and can be sold for $30. Both processes are fully automated, and variable overhead cost is charged at the rate of $40 per hour. (2.4)

2-37. The speed of your automobile has a huge effect on fuel consumption. Traveling at 65 miles per hour (mph) instead of 55 mph can consume almost 20% more fuel. As a general rule, for every mile per hour over 55 you lose 2% in fuel economy. For example, if your automobile gets 30 miles per gallon at 55 mph, the fuel consumption is 21 miles per gallon at 70 mph.

If you take a 400-mile trip and your average speed is 80 mph rather than the posted speed limit of 70 mph, what is the extra cost of fuel if gasoline costs $3.00 per gallon? Your car gets 30 miles per gallon (mpg) at 60 mph. (2.4)

2-38. The following results were obtained after analyzing the operational effectiveness of a production machine at two different speeds:

<table>
<thead>
<tr>
<th>Speed</th>
<th>Output (pieces per hour)</th>
<th>Time between tool grinds (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>400</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>540</td>
<td>10</td>
</tr>
</tbody>
</table>

A set of unsharpened tools costs $1,000 and can be ground 20 times. The cost of each grinding is $25. The time required to change and reset the tools is 1.5 hours, and such changes are made by a tool-setter who is paid $18/hour. The production machine operator is paid $15/hour, including the time that the machine is down for tool sharpening. Variable overhead on the machine is charged at the rate of $25/hour, including tool-changing time. A fixed-size production run will be made (independent of machine speed). (2.4)

a. At what speed should the machine be operated to minimize the total cost per piece? State your assumptions.
b. What is the basic trade-off in this problem?

2-39. An automatic machine can be operated at three speeds, with the following results:

<table>
<thead>
<tr>
<th>Speed</th>
<th>Output (pieces per hour)</th>
<th>Time between tool grinds (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>400</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>480</td>
<td>12</td>
</tr>
<tr>
<td>C</td>
<td>540</td>
<td>10</td>
</tr>
</tbody>
</table>
PROBLEMS

A set of unsharpened tools costs $500 and can be ground 20 times. The cost of each grinding is $25. The time required to change and reset the tools is 1.5 hours, and such changes are made by a tool-setter who is paid $8.00 per hour. Variable overhead on the machine is charged at the rate of $3.75 per hour, including tool-changing time. At which speed should the machine be operated to minimize total cost per piece? The basic trade-off in this problem is between the rate of output and tool usage. (2.4)

2-40. A company is analyzing a make-versus-purchase situation for a component used in several products, and the engineering department has developed these data:

Option A: Purchase 10,000 items per year at a fixed price of $8.50 per item. The cost of placing the order is negligible according to the present cost accounting procedure.

Option B: Manufacture 10,000 items per year, using available capacity in the factory. Cost estimates are direct materials = $5.00 per item and direct labor = $1.50 per item. Manufacturing overhead is allocated at 200% of direct labor (= $3.00 per item).

Based on these data, should the item be purchased or manufactured? (2.4)

2-41. A national car rental agency asks, “Do you want to bring back the economy-class car full of gas or with an empty tank? If we fill up the tank for you, we’ll charge you $2.70 per gallon, which is 30 cents less than the local price for gasoline.” Which choice should you make? State your assumptions. (2.4)

2-42. One method for developing a mine containing an estimated 100,000 tons of ore will result in the recovery of 62% of the available ore deposit and will cost $23 per ton of material removed. A second method of development will recover only 50% of the ore deposit, but it will cost only $15 per ton of material removed. Subsequent processing of the removed ore recovers 300 pounds of metal from each ton of processed ore and costs $40 per ton of ore processed. The recovered metal can be sold for $0.80 per pound. Which method for developing the mine should be used if your objective is to maximize total profit from the mine? (2.4)

2-43. Ocean water contains 0.9 ounce of gold per ton. Method A costs $220 per ton of water processed and will recover 85% of the metal. Method B costs $160 per ton of water processed and will recover 65% of the metal. The two methods require the same investment and are capable of producing the same amount of gold each day. If the extracted gold can be sold for $350 per ounce, which method of extraction should be used? Assume that the supply of ocean water is unlimited. Work this problem on the basis of profit per ounce of gold extracted. (2.4)

2-44. Which of the following statements are true and which are false? (all sections)

a. Working capital is a variable cost.
b. The greatest potential for cost savings occurs in the operation phase of the life cycle.
c. If the capacity of an operation is significantly changed (e.g., a manufacturing plant), the fixed costs will also change.
d. A noncash cost is a cash flow.
e. Goods and services have utility because they have the power to satisfy human wants and needs.
f. The demand for necessities is more inelastic than the demand for luxuries.
g. Indirect costs can normally be allocated to a specific output or work activity.
h. Present economy studies are often done when the time value of money is not a significant factor in the situation.
i. Overhead costs normally include all costs that are not direct costs.
j. Optimal volume (demand) occurs when total costs equal total revenues.
k. Standard costs per unit of output are established in advance of actual production or service delivery.
l. A related sunk cost will normally affect the prospective cash flows associated with a situation.
m. The life cycle needs to be defined within the context of the specific situation.
n. The greatest commitment of costs occurs in the acquisition phase of the life cycle.
o. High breakeven points in capital intensive industries are desirable.
p. The fixed return on borrowed capital (i.e., interest) is more risky than profits paid to equity investors (i.e., stockholders) in a firm.
q. There is no $D^*$ for this Scenario 1 situation: $p = 40 - 0.2D$ and $C_T = $100 + $30D$.
r. Most decisions are based on differences that are perceived to exist among alternatives.
s. A nonrefundable cash outlay (e.g., money spent on a passport) is an example of an opportunity cost.

2-45. A hot water leak in one of the faucets of your apartment can be very wasteful. A continuous leak of one quart per hour (a “slow” leak) at 155°F causes a loss of 1.75 million Btu per year. Suppose your water is heated with electricity. (2.3)

a. How many pounds of coal delivered to your electric utility does this leak equate to if one pound of coal contains 12,000 Btu and the boiler combustion process and water distribution system have an overall efficiency of 30%?

b. If a pound of coal produces 1.83 pounds of CO₂ during the combustion process, how much extra carbon dioxide does the leaky faucet produce in a year?

2-46. Extended Learning Exercise The student chapter of the American Society of Mechanical Engineers is planning a six-day trip to the national conference in Albany, NY. For transportation, the group will rent a car from either the State Tech Motor Pool or a local car dealer. The Motor Pool charges $0.36 per mile, has no daily fee, and the motor pool pays for the gas. The car dealer charges $30 per day and $0.20 per mile, but the group must pay for the gas. The car’s fuel rating is 20 miles per gallon, and the price of gas is estimated to be $2.00 per gallon. (2.2)

a. At what point, in miles, is the cost of both options equal?

b. The car dealer has offered a special student discount and will give the students 100 free miles per day. What is the new breakeven point?

c. Suppose now that the Motor Pool reduces its all-inclusive rate to $0.34 per mile and that the car dealer increases its rate to $30 per day and $0.28 per mile. In this case, the car dealer wants to encourage student business, so he offers 900 free miles for the entire six-day trip. He claims that if more than 750 miles are driven, students will come out ahead with one of his rental cars. If the students anticipate driving 2,000 miles (total), from whom should they rent a car? Is the car dealer’s claim entirely correct?

2-47. Web Exercise Home heating accounts for approximately one-third of energy consumption in a typical U.S. household. Despite soaring prices of oil, coal, and natural gas, one can make his/her winter heating bill noninflationary by installing an ultraconvenient corn burning stove that costs in the neighborhood of $2,400. That’s right—a small radiant-heating stove that burns corn and adds practically nothing to global warming or air pollution can be obtained through www.magnumfireplace.com. Its estimated annual savings in fuel is $300 in a regular U.S. farming community.

Conduct research on this means of home heating by accessing the above Web site. Do the annual savings you determine in your locale for a 2,000-square foot ranch-style house more than offset the cost of installing and maintaining a corn-burning stove? What other factors besides dollars might influence your decision to use corn for your home heating requirements? Be specific with your suggestions. (2.4)

Spreadsheet Exercises

2-48. Refer to Example 2-4. If your focus was on reducing expenses, would it be better to reduce the fixed cost (B1) or variable cost (B2) component? What is the effect of a +/− 10% change in both of these factors? (2.2)

2-49. Refer to Example 2-10. A competitor of your current blade supplier would like you to switch to their new model, which costs $5 less than your current supplier. After a long lunch, it comes out that this new blade has a life that is two cycles less than the current blade. Is the new blade a good deal? Remember that our current focus is on an unlimited amount of work. (2.4)

2-50. Refer to Example 2-11. Confirm that for any limited job size (in required board-feet), the cost of operating the planer at 5,000 ft/min is less than operating the planer at 6,000 ft/min. Why? (2.4)

2-51. Refer to Example 2-7. If the average inside temperature of this house in Virginia is increased from 65°F to 72°F, what is the most economical insulation amount? Assume that 100,000,000 Btu are lost with no insulation when the thermostat is set at 65°F. The cost of electricity is now $0.086 per kWh. In addition, the cost of insulation has increased by 50%. Develop a spreadsheet to solve this problem. (2.3)
Case Study Exercises

2-52. What are the key factors in this analysis, and how would your decision change if the assumed value of these factors changes? For example, what impact does rising fuel costs have on this analysis? Or, what if studies have shown that drivers can expect to avoid at least one accident every 10 years due to daytime use of headlights? (2.5)

2-53. Visit your local car dealer (either in person or online) to determine the cost of the daytime running lights option. How many accidents (per unit time) would have to be avoided for this option to be cost effective? (2.5)

FE Practice Problems

A company has determined that the price and the monthly demand of one of its products are related by the equation

\[ D = \sqrt{400 - p} \]

where \( p \) is the price per unit in dollars and \( D \) is the monthly demand. The associated fixed costs are $1,125/month, and the variable costs are $100/unit. Use this information to answer Problems 2-54 and 2-55. Select the closest answer. (2.2)

2-54. What is the optimal number of units that should be produced and sold each month?

(a) 10 units  
(b) 15 units  
(c) 20 units  
(d) 25 units

2-55. Which of the following values of \( D \) represents the breakeven point?

(a) 10 units  
(b) 15 units  
(c) 20 units  
(d) 25 units

A manufacturing company leases a building for $100,000 per year for its manufacturing facilities. In addition, the machinery in this building is being paid for in installments of $20,000 per year. Each unit of the product produced costs $15 in labor and $10 in materials. The product can be sold for $40. Use this information to answer Problems 2-56 through 2-58. Select the closest answer. (2.2)

2-56. How many units per year must be sold for the company to breakeven?

(a) 4,800  
(b) 3,000  
(c) 8,000  
(d) 6,667  
(e) 4,000

2-57. If 10,000 units per year are sold, what is the annual profit?

(a) $280,000  
(b) $50,000  
(c) $150,000  
(d) $-50,000  
(e) $30,000

2-58. If the selling price is lowered to $35 per unit, how many units must be sold each year for the company to earn a profit of $60,000 per year?

(a) 12,000  
(b) 10,000  
(c) 16,000  
(d) 18,000  
(e) 5,143

2-59. A recent engineering graduate was given the job of determining the best production rate for a new type of casting in a foundry. After experimenting with many combinations of hourly production rates and total production cost per hour, he summarized his findings in Table I. (See Table P2-59.) The engineer then talked to the firm’s marketing specialist, who provided estimates of selling price per casting as a function of production output (see Table II on page 62). There are 8,760 hours in a year. What production rate would you recommend to maximize total profits per year? (2.3)

(a) 100  
(b) 200  
(c) 300  
(d) 400  
(e) 500

2-60. A manufacturer makes 7,900,000 memory chips per year. Each chip takes 0.4 minutes of direct labor at the rate of $8 per hour. The overhead costs are estimated at $11 per direct labor hour. A new process will reduce the unit production time by 0.01 minutes. If the overhead cost will be reduced by $5.50 for each hour by which total direct hours are reduced, what is the maximum amount you will pay for the new process? Assume that the new process must pay for itself by the end of the first year. (2.4)

(a) $25,017  
(b) $1,066,500  
(c) $10,533  
(d) $17,775  
(e) $711,000
2-A Accounting Fundamentals

Accounting is often referred to as the language of business. Engineers should make serious efforts to learn about a firm’s accounting practice so that they can better communicate with top management. This section contains an extremely brief and simplified exposition of the elements of financial accounting in recording and summarizing transactions affecting the finances of the enterprise. These fundamentals apply to any entity (such as an individual or a corporation) called here a firm.

2.A.1 The Accounting Equation

All accounting is based on the fundamental accounting equation, which is

\[ \text{Assets} = \text{liabilities} + \text{owners' equity}, \]  

(2-A-1)

where assets are those things of monetary value that the firm possesses, liabilities are those things of monetary value that the firm owes, and owners’ equity is the worth of what the firm owes to its stockholders (also referred to as equities, net worth, etc.). For example, typical accounts in each term of Equation (2-A-1) are as follows:

\[
\begin{array}{ccc}
\text{Asset Accounts} & = & \text{Liability Accounts} + \text{Owner's Equity Accounts} \\
\text{Cash} & \text{Short-term debt} & \text{Capital stock} \\
\text{Receivables} & \text{Payables} & \text{Retained earnings (income retained in} \\
\text{Inventories} & \text{Long-term debt} & \text{the firm)} \\
\text{Equipment} & & \\
\text{Buildings} & & \\
\text{Land} & & 
\end{array}
\]

The fundamental accounting equation defines the format of the balance sheet, which is one of the two most common accounting statements and which shows the financial position of the firm at any given point in time.

Another important, and rather obvious, accounting relationship is

\[ \text{Revenues} - \text{expenses} = \text{profit (or loss)}, \]  

(2-A-2)

This relationship defines the format of the income statement (also commonly known as a profit-and-loss statement), which summarizes the revenue and expense results of operations over a period of time. Equation (2-A-1) can be expanded to take into account profit as defined in Equation (2-A-2):

\[ \text{Assets} = \text{liabilities} + (\text{beginning owners' equity} + \text{revenue} - \text{expenses}). \]  

(2-A-3)
Profit is the increase in money value (not to be confused with cash) that results from a firm’s operations and is available for distribution to stockholders. It therefore represents the return on owners’ invested capital.

A useful analogy is that a balance sheet is like a snapshot of the firm at an instant in time, whereas an income statement is a summarized moving picture of the firm over an interval of time. It is also useful to note that revenue serves to increase owners’ interests in a firm, but an expense serves to decrease the owners’ equity amount for a firm.

To illustrate the workings of accounts in reflecting the decisions and actions of a firm, suppose that an individual decides to undertake an investment opportunity and the following sequence of events occurs over a period of one year:

1. Organize XYZ firm and invest $3,000 cash as capital.
2. Purchase equipment for a total cost of $2,000 by paying cash.
3. Borrow $1,500 through a note to the bank.
4. Manufacture year’s supply of inventory through the following:
   (a) Pay $1,200 cash for labor.
   (b) Incur $400 accounts payable for material.
   (c) Recognize the partial loss in value (depreciation) of the equipment amounting to $500.
5. Sell on credit all goods produced for year, 1,000 units at $3 each. Recognize that the accounting cost of these goods is $2,100, resulting in an increase in equity (through profits) of $900.
6. Collect $2,200 of accounts receivable.
7. Pay $300 of accounts payable and $1,000 of bank note.

A simplified version of the accounting entries recording the same information in a format that reflects the effects on the fundamental accounting equation (with a “+” denoting an increase and a “−” denoting a decrease) is shown in Figure 2-A-1. A summary of results is shown in Figure 2-A-2.

It should be noted that the profit for a period serves to increase the value of the owners’ equity in the firm by that amount. Also, it is significant that the net cash flow from operation of $700 (= $2,200 − $1,200 − $300) is not the same as profit. This amount was recognized in transaction 4(c), in which capital consumption (depreciation) for equipment of $500 was declared. Depreciation serves to convert part of an asset into an expense, which is then reflected in a firm’s profits, as seen in Equation (2-A-2). Thus, the profit was $900, or $200 more than the net cash flow. For our purposes, revenue is recognized when it is earned, and expenses are recognized when they are incurred.

One important and potentially misleading indicator of after-the-fact financial performance that can be obtained from Figure 2-A-2 is “annual rate of return.” If the invested capital is taken to be the owners’ (equity) investment, the annual rate of return at the end of this particular year is $900/$3,900 = 23%.

Financial statements are usually most meaningful if figures are shown for two or more years (or other reporting periods such as quarters or months) or for two or more individuals or firms. Such comparative figures can be used to reflect trends or financial indications that are useful in enabling investors and management to determine the effectiveness of investments after they have been made.

2.A.2 Cost Accounting

Cost accounting, or management accounting, is a phase of accounting that is of particular importance in engineering economic analysis because it is concerned principally with
<table>
<thead>
<tr>
<th>Account</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Balances at End of Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>+$3,000</td>
<td>−$2,000</td>
<td>+$1,500</td>
<td>−$1,200</td>
<td>+$2,200</td>
<td>−$1,300</td>
<td>+$2,200</td>
<td>$4,500</td>
</tr>
<tr>
<td>Accounts receivable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$800</td>
</tr>
<tr>
<td>Inventory</td>
<td></td>
<td>+2,100</td>
<td>−2,100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>Equipment</td>
<td>+2,000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+1,500</td>
</tr>
<tr>
<td><strong>Equals</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$4,500</td>
</tr>
<tr>
<td>Accounts payable</td>
<td></td>
<td>+400</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>−300</td>
</tr>
<tr>
<td>Bank note</td>
<td></td>
<td>+1,500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+100</td>
</tr>
<tr>
<td><strong>Plus</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$4,500</td>
</tr>
<tr>
<td>Owners' equity</td>
<td>Equity</td>
<td>+3,000</td>
<td></td>
<td></td>
<td></td>
<td>+900</td>
<td></td>
<td>+3,900</td>
</tr>
</tbody>
</table>

Figure 2-A-1  Accounting Effects of Transactions: XYZ Firm
XYZ Firm Balance Sheet
As of December 31, 2006

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities and Owners’ Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash $2,200</td>
<td>Bank note $500</td>
</tr>
<tr>
<td>Accounts receivable 800</td>
<td>Accounts payable 100</td>
</tr>
<tr>
<td>Equipment 1,500</td>
<td>Equity 3,900</td>
</tr>
<tr>
<td><strong>Total $4,500</strong></td>
<td><strong>Total $4,500</strong></td>
</tr>
</tbody>
</table>

XYZ Firm Income Statement
for Year Ending December 31, 2006

<table>
<thead>
<tr>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating revenues (Sales) $3,000</td>
</tr>
<tr>
<td>Operating costs (Inventory depleted)</td>
</tr>
<tr>
<td>Labor $1,200</td>
</tr>
<tr>
<td>Material 400</td>
</tr>
<tr>
<td>Depreciation 500</td>
</tr>
<tr>
<td><strong>Total $2,100</strong></td>
</tr>
<tr>
<td>Net income (Profits)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Figure 2-A-2  Balance Sheet and Income Statement Resulting from Transactions Shown in Figure 2-A-1

decision making and control in a firm. Consequently, cost accounting is the source of much of the cost data needed in making engineering economy studies. Modern cost accounting may satisfy any or all of the following objectives:

1. Determination of the actual cost of products or services
2. Provision of a rational basis for pricing goods or services
3. Provision of a means for allocating and controlling expenditures
4. Provision of information on which operating decisions may be based and by means of which operating decisions may be evaluated

Although the basic objectives of cost accounting are simple, the exact determination of costs usually is not. As a result, some of the procedures used are arbitrary devices that make it possible to obtain reasonably accurate answers for most cases but that may contain a considerable percentage of error in other cases, particularly with respect to the actual cash flow involved.

2.A.3 The Elements of Cost

One of the first problems in cost accounting is that of determining the elements of cost that arise in the production of a product or the rendering of a service. A study of how these costs occur gives an indication of the accounting procedure that must be established to give satisfactory cost information. Also, an understanding of the procedure that is used to account for these costs makes it possible to use them more intelligently.
From an engineering and managerial viewpoint in manufacturing enterprises, it is common to consider the general elements of cost to be direct materials, direct labor, and overhead. Such terms as burden and indirect costs are often used synonymously with overhead, and overhead costs are often divided into several subcategories.

Ordinarily, the materials that can be conveniently and economically charged directly to the cost of the product are called direct materials. Several guiding principles are used when we decide whether a material is classified as a direct material. In general, direct materials should be readily measurable, be of the same quantity in identical products, and be used in economically significant amounts. Those materials that do not meet these criteria are classified as indirect materials and are a part of the charges for overhead. For example, the exact amount of glue and sandpaper used in making a chair would be difficult to determine. Still more difficult would be the measurement of the exact amount of coal that was used to produce the steam that generated the electricity that was used to heat the glue. Some reasonable line must be drawn beyond which no attempt is made to measure directly the material that is used for each unit of production.

Labor costs also are ordinarily divided into direct and indirect categories. Direct labor costs are those that can be conveniently and easily charged to the product or service in question. Other labor costs, such as for supervisors, material handlers, and design engineers, are charged as indirect labor and are thus included as part of overhead costs. It is often imperative to know what is included in direct labor and direct material cost data before attempting to use them in engineering economy studies.

In addition to indirect materials and indirect labor, there are numerous other cost items that must be incurred in the production of products or the rendering of services. Property taxes must be paid; accounting and personnel departments must be maintained; buildings and equipment must be purchased and maintained; supervision must be provided. It is essential that these necessary overhead costs be attached to each unit produced in proper proportion to the benefits received. Proper allocation of these overhead costs is not easy, and some factual, yet reasonably simple, method of allocation must be used.

As might be expected, where solutions attempt to meet conflicting requirements such as exist in overhead-cost allocation, the resulting procedures are empirical approximations that are accurate in some cases and less accurate in others.

There are many methods of allocating overhead costs among the products or services produced. The most commonly used methods involve allocation in proportion to direct labor cost, direct labor hours, direct materials cost, sum of direct labor and direct materials cost, or machine hours. In these methods, it is necessary to estimate what the total overhead costs will be if standard costs are being determined. Accordingly, total overhead costs are customarily associated with a certain level of production, which is an important condition that should always be remembered when dealing with unit-cost data. These costs can be correct only for the conditions for which they were determined.

To illustrate one method of allocation of overhead costs, consider the method that assumes that overhead is incurred in direct proportion to the cost of direct labor used. With this method, the overhead rate (overhead per dollar of direct labor) and the overhead cost per unit would respectively be

$$\text{Overhead rate} = \frac{\text{total overhead in dollars for period}}{\text{direct labor in dollars for period}}$$

$$\text{Overhead cost/unit} = \text{overhead rate} \times \text{direct labor cost/unit}. \quad (2-A-4)$$

Suppose that for a future period (say, a quarter), the total overhead cost is expected to be $100,000 and the total direct labor cost is expected to be $50,000. From this,
The overhead rate = $100,000 / $50,000 = $2 per dollar of direct labor cost. Suppose further that for a given unit of production (or job) the direct labor cost is expected to be $60. From Equation (2-A-4), the overhead cost for the unit of production would be $60 × 2 = $120.

This method obviously is simple and easy to apply. In many cases, it gives quite satisfactory results. However, in many other instances, it gives only very approximate results because some items of overhead, such as depreciation and taxes, have very little relationship to labor costs. Quite different total costs may be obtained for the same product when different procedures are used for the allocation of overhead costs. The magnitude of the difference will depend on the extent to which each method produces or fails to produce results that realistically capture the facts.

2.A.4 Cost Accounting Example

This relatively simple example involves a job-order system in which costs are assigned to work by job number. Schematically, this process is illustrated in the following diagram:

Costs are assigned to jobs in the following manner:

1. Raw materials attach to jobs via material requisitions.
2. Direct labor attaches to jobs via direct labor tickets.
3. Overhead cannot be attached to jobs directly but must have an allocation procedure that relates it to one of the resource factors, such as direct labor, which is already accumulated by the job.

Consider how an order for 100 tennis rackets accumulates costs at the Bowling Sporting Goods Company:

<table>
<thead>
<tr>
<th></th>
<th>100 tennis rackets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job #161</td>
<td></td>
</tr>
<tr>
<td>Labor rate</td>
<td>$7 per hour</td>
</tr>
<tr>
<td>Leather</td>
<td>50 yards at $2 per yard</td>
</tr>
<tr>
<td>Gut</td>
<td>300 yards at $0.50 per yard</td>
</tr>
<tr>
<td>Graphite</td>
<td>180 pounds at $3 per pound</td>
</tr>
<tr>
<td>Labor hours for the job</td>
<td>200 hours</td>
</tr>
<tr>
<td>Total annual factory overhead costs</td>
<td>$600,000</td>
</tr>
<tr>
<td>Total annual direct labor hours</td>
<td>200,000 hours</td>
</tr>
</tbody>
</table>
The three major costs are now attached to the job. Direct labor and material expenses are straightforward:

<table>
<thead>
<tr>
<th>Job #161</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct labor</td>
<td>$1,400</td>
</tr>
<tr>
<td>Direct material</td>
<td>$100</td>
</tr>
<tr>
<td>Leather</td>
<td>$2</td>
</tr>
<tr>
<td>Gut</td>
<td>$0.5</td>
</tr>
<tr>
<td>Graphite</td>
<td>$3</td>
</tr>
</tbody>
</table>

Prime costs (direct labor + direct materials) $2,190

Notice that this cost is not the total cost. We must somehow find a way to attach (allocate) factory costs that cannot be directly identified to the job but are nevertheless involved in producing the 100 rackets. Costs such as the power to run the graphite molding machine, the depreciation on this machine, the depreciation of the factory building, and the supervisor’s salary constitute overhead for this company. These overhead costs are part of the cost structure of the 100 rackets but cannot be directly traced to the job. For instance, do we really know how much machine obsolescence is attributable to the 100 rackets? Probably not. Therefore, we must allocate these overhead costs to the 100 rackets by using the overhead rate determined as follows:

Overhead rate = \( \frac{600,000}{200,000} \) = $3 per direct labor hour.

This means that $600 ($3 × 200) of the total annual overhead cost of $600,000 would be allocated to Job #161. Thus, the total cost of Job #161 would be as follows:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct labor</td>
<td>$1,400</td>
</tr>
<tr>
<td>Direct materials</td>
<td>790</td>
</tr>
<tr>
<td>Factory overhead</td>
<td>600</td>
</tr>
</tbody>
</table>

$2,790

The cost of manufacturing each racket is thus $27.90. If selling expenses and administrative expenses are allocated as 40% of the cost of goods sold, the total expense of a tennis racket becomes \( 1.4 \times 27.90 \) = $39.06.

**Problems**

**2-A-1.** Jill Smith opens an apartment-locator business near a college campus. She is the sole owner of the proprietorship, which she names Campus Apartment Locators. During the first month of operations, July 2007, she engages in the following transactions:

a. Smith invests $35,000 of personal funds to start the business.

b. She purchases on account office supplies costing $350.

c. Smith pays cash of $30,000 to acquire a lot next to the campus. She intends to use the land as a future building site for her business office.

d. Smith locates apartments for clients and receives cash of $1,900.

APPENDIX 2-A / PROBLEMS

TABLE P2-A-2  Data for Problem 2-A-2

<table>
<thead>
<tr>
<th>Assets</th>
<th>= Liabilities</th>
<th>+ Owner’s Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounts Accounts Accounts</td>
<td>Accounts Payable</td>
<td>Daniel Peavy,</td>
</tr>
<tr>
<td>Cash + Receivable + Supplies + Land</td>
<td>+</td>
<td>Capital</td>
</tr>
</tbody>
</table>

| Bal.  | 3,240 | 24,100 | 5,400 | 23,660 |

1. Paid office rent, $1,200.
2. Paid advertising, $660.
i. Peavy sold supplies to another interior designer for $80 cash, which was the cost of the supplies.

Required

(a) Analyze the effects of the preceding transactions on the accounting equation of Peavy Design. Adapt the format of Figure 2-A-1.

(b) Prepare the income statement of Peavy Design for the month ended May 31, 2007. List expenses in decreasing order by amount.

(c) Prepare the balance sheet of Peavy Design at May 31, 2007.

2-A-3. Lubbock Engineering Consultants is a firm of professional civil engineers. It mostly does surveying jobs for the heavy construction industry throughout Texas. The firm obtains its jobs by giving fixed-price quotations, so profitability depends on the ability to predict the time required for the various subtasks on the job. (This situation is similar to that in the auditing profession, where times are budgeted for such audit steps as reconciling cash and confirming accounts receivable.)

A client may be served by various professional staff, who hold positions in the hierarchy from partners to managers to senior engineers to assistants. In addition, there are secretaries and other employees.

Lubbock Engineering has the following budget for 2008:

<table>
<thead>
<tr>
<th></th>
<th>$3,600,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation of professional staff</td>
<td>$3,600,000</td>
</tr>
<tr>
<td>Other costs</td>
<td>1,449,000</td>
</tr>
<tr>
<td>Total budgeted costs</td>
<td>$5,049,000</td>
</tr>
</tbody>
</table>

Each professional staff member must submit a weekly time report, which is used for charging hours to a client job-order record. The time report has seven columns, one for each day of the week. Its rows are as follows:

- Chargeable hours
  - Client 156
  - Client 183
  - Etc.

- Nonchargeable hours
  - Attending seminar on new equipment
  - Unassigned time
  - Etc.

In turn, these time reports are used for charging hours and costs to the client job-order records. The managing partner regards these job records as absolutely essential for measuring the profitability of various jobs and for providing an “experience base for improving predictions on future jobs.”

a. The firm applies overhead to jobs at a budgeted percentage of the professional compensation charged directly to the job (“direct labor”). For all categories of professional personnel, chargeable hours average 85% of available hours. Nonchargeable hours are regarded as additional overhead. What is the overhead rate as a percentage of “direct labor,” the chargeable professional compensation cost?

b. A senior engineer works 48 weeks per year, 40 hours per week. His compensation is $60,000. He has worked on two jobs during the past week, devoting 10 hours to Job 156 and 30 hours to Job 183. How much cost should be charged to Job 156 because of his work there?