

4. Number Systems

OBJECTIVES

After reading this unit, you will:

- Learn about sets of real numbers and the properties of real numbers.
-

Calculus is based on the real number system. We start with the following notation for subsets of the set of real numbers and for the set of real numbers itself:

- \mathbb{N} : **Natural numbers** $\{1, 2, 3, 4, \dots\}$

- \mathbb{Z} : **Integers** $\{\dots - 3, -2, -1, 0, 1, 2, 3, \dots\}$

- \mathbb{Q} : **Rational Numbers**: a number that can be written as the quotient of two integers. That is,

$$r \in \mathbb{Q} \Leftrightarrow r = \frac{m}{n}, m, n \in \mathbb{Z}, n \neq 0$$

Any repeating or terminating decimal is a rational number. For example,

$$\frac{1}{2}, \quad 0 = \frac{0}{1}, \quad 5 = \frac{5}{1}, \quad \frac{-123}{452}, \quad 0.822 = \frac{822}{1000}$$

- **Irrational Numbers**: none of the above i.e. non-terminating and non-repeating decimals are irrational.

For example,

$$\begin{aligned} & \sqrt{2}, \sqrt{10}, -\sqrt{5} \\ \pi &= 3.14159265358979\dots, \quad e = 2.71828182845\dots, \\ & 1.01001000100001\dots \end{aligned}$$

- \mathbb{R} : Real Numbers - all of the above i.e. the union of the sets of rational and irrational numbers.

It is interesting to note the juxtaposition of the various sets in a diagram:

INSERT FIGURE 3.1 FROM TEXT

Properties of Real Numbers

Let $a, b, c, d \in \mathbb{R}$. Then:

(i)

$$\begin{aligned} -(-a) &= a, \\ -(+a) &= -a, \\ (-a)(-b) &= ab, \\ (-a) + (-b) &= -(a + b) \end{aligned}$$

(ii) If $b, d \neq 0$,

$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \Leftrightarrow ad = bc, \\ \frac{a}{b} + \frac{c}{b} &= \frac{a + c}{b}, \\ \left(\frac{a}{b}\right) \left(\frac{c}{d}\right) &= \frac{ac}{bd}, \\ \frac{ad}{bd} &= \frac{a}{b}, \end{aligned}$$

(iii)

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}, \quad b, d \neq 0$$

(This is called *taking a common denominator*)

Be Careful!

(a) Division by zero is undefined. For example,

$$\frac{3}{0}, \frac{-1}{0}, \frac{0}{0} \text{ are meaningless}$$

but zero itself can be divided by any non-zero number to give zero again i.e.

$$\frac{0}{2} = \frac{0}{-1} = \frac{0}{\frac{1}{2}} = 0$$

(b)

$$\frac{a}{b+d} \neq \frac{a}{b} + \frac{a}{d}$$

e.g. $\frac{1}{2+4} \neq \frac{1}{2} + \frac{1}{4}$

As simple as they may seem, these are among the two most common mistakes made by first year engineering students.